

Quiz 2  
Empirical Bayes Estimation and Random Intercept Linear Models

From Tuesday's lecture on the two-stage model and empirical bayes estimation, answer the following questions:

1. In a two-stage model, the major sources of variation in an estimate of a regression parameter (e.g. log relative risk) are (check all that apply):
  - (a). statistical error the arises from imprecision in the finite set of measurements
  - (b). Bayesian error
  - (c). conjugate distribution error
  - (d). natural variation in the true parameter values
  - (e). stochastic correspondence deviations
  
2. In estimating the average parameter value (here, log relative risk) across cities, we should weight the city-specific estimates: (choose best answer):
  - (a). inversely proportional to the standard error
  - (b). proportional to the standard error
  - (c). inversely proportional to the statistical variance
  - (d). proportional to the statistical variance
  - (e). inversely proportional to the sum of the statistical and natural variance
  
3. When the statistical variance is small relative to the natural variance, we estimate each city's parameter value by: (choose best answers):
  - (a). the un-weighted average of all the city-specific estimates
  - (b). that city's maximum likelihood estimate
  - (c). the weighted average of all the city-specific estimates
  - (d). a linear combination of the city-specific mle and the overall un-weighted average
  - (e). a linear combination of the city-specific mle and the overall weighted average
  
4. Relative to the mle, the empirical Bayes estimate for a city's parameter (e.g. log relative risk) is: (check all correct answers)
  - (a). is shrunk toward the overall estimate
  - (b). is more biased
  - (c). is more precise
  - (d). is less biased
  - (e). is less precise

From Wednesday's lecture on linear random intercept models, answer the following questions:

5. In the linear random intercept model example from the lecture, we define two sources of variation from measurements of the guinea pigs. These are:

- (a). random variation in the outcome of interest measured within the same guinea pig over time
- (b). measurement error in any weight measurement
- (c). natural heterogeneity in the guinea pigs which may represent genetic variation
- (d). statistical variation in the measurement of the rate of change of weight with time

6. When you specify a linear random intercept model, what type of correlation structure are you defining?

- (a). an independence structure, i.e. no correlation
- (b). an auto-regressive correlation structure, observations within units become less correlated over time
- (c). an exchangeable correlation structure, the correlation is the same between any two measurements from the same unit (time is exchangeable)
- (d). no correlation structure, we are estimating the ratio of natural variance to total variance.

7. Consider the following design: You have a random sample of 50 hospitals (indexed by  $i$ ) and within each hospital you sample a varying number of surgery patients (indexed by  $j = 1, 2, \dots, n_i$ ). The goal of the study is to model the average LOS as a function of a patient severity score. Suppose for now that we can model LOS as a linear variable (although it is really a count!). You specify the following model:

$$E(\text{LOS}_{ij}) = b_0 + b_1 \text{severity}_{ij} + u_i + e_{ij}$$

where  $u_i \sim \text{Normal}(0, \tau^2)$  and  $e_{ij} \sim \text{Normal}(0, \sigma^2)$ .

In words, interpret  $\tau^2$  and  $\sigma^2$ .

This is a linear random intercept model, which induces an exchangeable correlation structure among patients from the same hospital. In words explain what this means in this setting.