Interpretations in Hierarchical Logistic Regression Models

BIO 656

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Suppose our levels are school (i), classroom(j), and child (k). The model is given by:

$$\begin{aligned} logit\{P(Y_{ijk} = 1 | U_i, U_{ij}, x_{ijk})\} &= \beta_0 + \beta_1 x_{ijk} + U_i + U_{ij} \\ U_i \sim N(0, \sigma^2), \qquad & U_{ij} \sim N(0, \tau^2) \end{aligned}$$

1.

$$\beta_0 = logit\{P(Y_{ijk} = 1 | U_i = 0, U_{ij} = 0, x_{ijk} = 0)\}$$

β_0 is the log-odds of the outcome for a child with $x_{ijk} = 0$ from the typical classroom ($U_{ij} = 0$) in the typical school ($U_i = 0$).

2.

$$\begin{aligned} U_i &= logit\{P(Y_{ijk} = 1 | U_i, U_{ij} = u, x_{ijk} = x)\} - logit\{P(Y_{ijk} = 1 | U_i = 0, U_{ij} = u, x_{ijk} = x)\} \\ &= (\beta_0 + \beta_1 x + U_i + u) - (\beta_0 + \beta_1 x + 0 + u) \end{aligned}$$

U_i is the difference between the log-odds of the outcome in school i and the log-odds of the outcome in the typical school ($U_i = 0$), comparing classrooms of equal random effect (holding U_{ij} constant) and holding x_{ijk} constant.

3.

$$\begin{aligned} U_{ij} &= logit\{P(Y_{ijk} = 1 | U_i = u, U_{ij}, x_{ijk} = x)\} - logit\{P(Y_{ijk} = 1 | U_i = u, U_{ij} = 0, x_{ijk} = x)\} \\ &= (\beta_0 + \beta_1 x + u + U_{ij}) - (\beta_0 + \beta_1 x + u + 0) \end{aligned}$$

U_{ij} is the difference between the log-odds of the outcome in classroom j within school i and the log-odds of the outcome in the typical classroom within school i ($U_{ij} = 0$) holding x_{ijk} constant.

4.

$$\begin{aligned} \beta_1 &= logit\{P(Y_{ijk} = 1 | U_i = u_1, U_{ij} = u_2, x_{ijk} = x + 1)\} - logit\{P(Y_{ijk} = 1 | U_i = u_1, U_{ij} = u_2, x_{ijk} = x)\} \\ &= (\beta_0 + \beta_1 (x + 1) + u_1 + u_2) - (\beta_0 + \beta_1 x + u_1 + u_2) \end{aligned}$$

β_1 is the difference between the log-odds of the outcome for children in the same classroom in the same school who have observed x values that differ by one unit.

5.

$$\sigma^2 \quad = \quad Var(U_i) \tag{1}$$

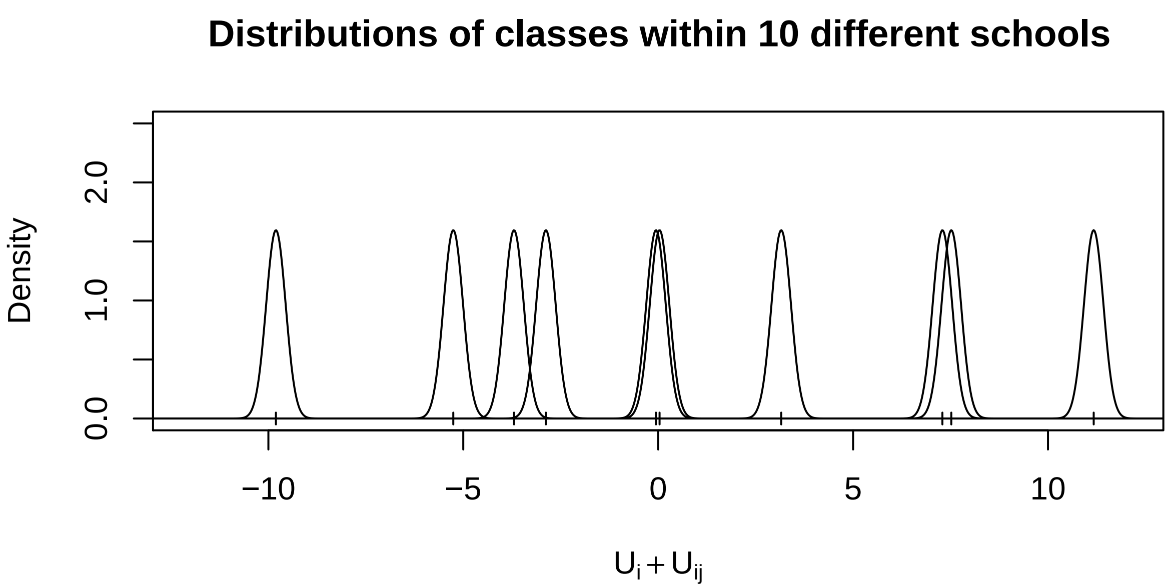
σ^2 is the variance of the random intercept for school. In other words, it describes the variability between schools after accounting for the differing values of the predictor for the children within schools.

6.

$$\tau^2 \quad = \quad Var(U_{ij}) \tag{2}$$

τ^2 is the variance of the random interecept for classroom. In other words, it describes the variability between classrooms within the same school after accounting for the differing values of the predictor for the children within those classrooms.

Let's quickly visualize the distributions of the random effects. Let's say we have 25 classsrooms in each of 10 schools. Then the distributions of the classes (after accounting for effects of the predictor) can be seen as 10 different normal distributions, each centered around the school-specific mean:



Each tick mark represents the school-specific mean, and the plotted distributions are the classrooms within each school. Therefore, σ^2 describes the variability between the tick marks, while τ^2 describes the variability within the plotted normal distributions. In this particular realization of the data, there is much more variability between schools ($\sigma^2 = 5$) than between classrooms within schools ($\tau^2 = 0.25$).