

A brief introduction to Mathematica

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Why Mathematica?

C	computational efficiency
Perl	text/data manipulation
R	interactive analyses

Mathematica symbolic algebra/calculus

Everything you could do back when you had just taken calculus, only **accurately**.

You might instead use **Maple**, but I have no experience with it.

Preliminaries

- Command-line version: type `math`
(I use this, and copy-and-paste from a text file.)
- GUI (with mathematica “notebooks”): type `mathematica`
- To exit: type `Quit`

First stuff

```
In[1] := 5^12  
Out[1] = 244140625
```

```
In[2] := %1 ^ (1/12)  
Out[2] = 5
```

```
In[3] := % + a  
Out[3] = 5 + a
```

```
In[4] := L = 3  
Out[4] = 3
```

```
In[5] := L  
Out[5] = 3
```

```
In[6] := L = 3;
```

```
In[7] := 20 L  
Out[7] = 60
```

```
In[8] := 2 m + 3 m  
Out[8] = 5 m
```

```
In[9] := %%  
Out[9] = 60
```

```
In[10] := %8  
Out[10] = 5 m
```

Help

?Factor*

?FactorInteger

??FactorInteger

?*Plot*

?@ (* defined objects *)

Buy a book, such as Abell & Braselton, *Mathematica by Example*, 3rd ed.

Use Google.

Packages

Sometimes, you need to load a separate package. I don't recall ever needing this.

```
In[1]:= GramSchmidt[{{1,1,0}, {0,2,1}, {1,0,3}}]
```

```
Out[1]= GramSchmidt[{{1, 1, 0}, {0, 2, 1}, {1, 0, 3}}]
```

```
In[2]:= Remove[GramSchmidt]
```

```
In[3]:= << LinearAlgebra`Orthogonalization`
```

```
In[4]:= GramSchmidt[{{1,1,0}, {0,2,1}, {1,0,3}}]
```

```
Out[4]= {{-----, -----, 0}, {-(-----), -----, -----},
          Sqrt[2]   Sqrt[2]           Sqrt[3]   Sqrt[3]   Sqrt[3]}
```

```
> {-----, -(-----), Sqrt[-]}
   Sqrt[6]   Sqrt[6]       3
```

A bit of notation

[] Arguments to functions

{ } Lists

[[]] Subsetting lists

Numbers

```
In[1]:= 5 * 10  
Out[1]= 50
```

```
In[2]:= 5 10  
Out[2]= 50
```

```
In[3]:= a10  
Out[3]= a10
```

```
In[4]:= a 10  
Out[4]= 10 a
```

```
In[5]:= 1/2 + 2/144
```

```
Out[5]=  $\frac{37}{72}$ 
```

```
In[6]:= 1/2 + 2.0/144  
Out[6]= 0.513889
```

```
In[7]:= Sqrt[27]  
Out[7]= 3 Sqrt[3]
```

```
In[8]:= Sqrt[27.0]  
Out[8]= 5.19615
```

```
In[9]:= N[Sqrt[27]]  
Out[9]= 5.19615
```

Constants

```
In[1]:= E
```

```
Out[1]= E
```

```
In[2]:= Pi
```

```
Out[2]= Pi
```

```
In[3]:= N[E, 25]
```

```
Out[3]= 2.718281828459045235360287
```

```
In[4]:= N[Pi, 100]
```

```
Out[4]= 3.1415926535897932384626433832795028841971693993751058209749445923078\  
> 16406286208998628034825342117068
```

Algebra

Expand, Factor, Together, Apart, Simplify, FullSimplify

```
In[1]:= Expand[ (x + 2y + z)^2 ]
```

```
Out[1]= x2 + 4 x y + 4 y2 + 2 x z + 4 y z + z2
```

```
In[2]:= Factor[ % ]
```

```
Out[2]= (x + 2 y + z)2
```

```
In[3]:= Together[ 1/(1+2x) - 2/(2+3x) ]
```

```
Out[3]= -( $\frac{x}{(1 + 2 x) (2 + 3 x)}$ )
```

```
In[4]:= Apart[ % ]
```

```
Out[4]=  $\frac{1}{1 + 2 x} - \frac{2}{2 + 3 x}$ 
```

Solving equations

Solve, NRoots

```
In[1]:= f = x^3 - 3 x^2 - 17x + 51;
```

```
In[2]:= soln = Solve[f == 0, x]
```

```
Out[2]= {{x -> 3}, {x -> -Sqrt[17]}, {x -> Sqrt[17]}}
```

```
In[3]:= f /. soln
```

```
Out[3]= {0, 0, 0}
```

```
In[4]:= NRoots[ f == 0, x ]
```

```
Out[4]= x == -4.12311 || x == 3. || x == 4.12311
```

A silly example

Take $V_g = a^2/2 + d^2/4$, $V_e = V_g(1 - h^2)/h^2$, and $a = 4d$.

Supposing $V_e = 1$ and $h^2 = 0.6$, solve for d .

```
In[1]:= Vg = a^2 / 2 + d^2 / 4;
```

```
In[2]:= Ve = Vg (1-hsq)/hsq;
```

```
In[3]:= a = 4d;
```

```
In[4]:= hsq = 6/10;
```

```
In[5]:= Solve[Ve == 1, d]
```

```
Out[5]= {{d -> -Sqrt[ $\frac{11}{11}$ ]}, {d -> Sqrt[ $\frac{11}{11}$ ]}}
```

```
In[6]:= N[ % ]
```

```
Out[6]= {{d -> -0.426401}, {d -> 0.426401}}
```

Solving systems

Suppose we have $2p_1 + 2p_2 = 1$ and $p_1 = (1 - r)p_1 + p_2/2$.

Solve for p_1 and p_2 .

```
In[1]:= eqn1 = 2 p1 + 2 p2 == 1;
```

```
In[2]:= eqn2 = p1 == (1-r) p1 + p2 / 2;
```

```
In[3]:= Solve[ {eqn1, eqn2}, {p1, p2}]
```

```
Out[3]= {{p1 ->  $\frac{1}{2(1+2r)}$ , p2 ->  $\frac{r}{1+2r}$ }}
```

A nonlinear example

Suppose $x^2 = 2y + 2$ and $x = y^2 + 1$.

Solve for x and y .

```
In[1]:= N [ Solve[ {x^2 == 2y + 2, x == y^2 + 1}, {x,y} ] ]
```

```
Out[1]= {{x -> 2., y -> 1.}, {x -> 1.1304, y -> -0.361103},
```

```
> {x -> -1.5652 - 1.04343 I, y -> -0.319448 + 1.63317 I},
```

```
> {x -> -1.5652 + 1.04343 I, y -> -0.319448 - 1.63317 I}}
```

Series

```
In[1]:= Sum[ Exp[-mu] mu^n / Factorial[n], {n, 0, Infinity} ]
```

```
Out[1]= 1
```

```
In[2]:= Sum[ n Exp[-mu] mu^n / Factorial[n], {n, 0, Infinity} ]
```

```
Out[2]= mu
```

```
In[3]:= Sum[ (n - mu)^2 Exp[-mu] mu^n / Factorial[n], {n, 0, Infinity} ]
```

```
Out[3]= mu
```

```
In[4]:= Sum[ p^k, {k, 0, n} ]
```

```
Out[4]= 
$$\frac{1 + n}{-1 + p}$$

```

```
In[5]:= Sum[ p^k, {k, 1, n} ]
```

```
Out[5]= 
$$\frac{p (-1 + p)^n}{-1 + p}$$

```

Limits

```
In[1]:= Limit[ Sin[x]/x, x -> 0 ]
```

```
Out[1]= 1
```

```
In[2]:= Limit[ 1/x, x -> Infinity ]
```

```
Out[2]= 0
```

```
In[3]:= Limit[ 1/x, x->0, Direction -> -1 ]
```

```
Out[3]= Infinity
```

```
In[4]:= Limit[ 1/x, x->0, Direction -> 1 ]
```

```
Out[4]= -Infinity
```

Integrals & derivatives

```
In[1]:= Integrate[ x^4 Cos[x], x ]
```

```
Out[1]= 4 x2 (-6 + x2) Cos[x] + (24 - 12 x2 + x4) Sin[x]
```

```
In[2]:= D[%, x]
```

```
Out[2]= 8 x2 Cos[x] + 4 (-6 + x2) Cos[x] + (24 - 12 x2 + x4) Cos[x] -  
> 4 x2 (-6 + x2) Sin[x] + (-24 x + 4 x3) Sin[x]
```

```
In[3]:= Simplify[%]
```

```
Out[3]= x4 Cos[x]
```

```
In[4]:= Integrate[ Exp[x], {x, -1, 1} ]
```

```
Out[4]=  $\frac{1}{E} - (-) + E$ 
```

```
In[5]:= Together[%]
```

```
Out[5]=  $\frac{-1 + E^2}{E}$ 
```

Another example

Consider $X_1, X_2, X_3 \sim \text{iid } N(\mu, \sigma^2)$.

Define $R = [X_{(2)} - X_{(1)}] / [X_{(3)} - X_{(1)}]$.

One can show that the density of R is $f(r) = \frac{3\sqrt{3}}{2\pi} \cdot \frac{1}{r^2 + r(1-r) + (1-r)^2}$

Find the cdf.

```
In[1]:= Integrate[ 3 Sqrt[3]/(2 Pi) / (r^2 + r(1-r) + (1-r)^2), r]
```

```
          -1 + 2 r
    3 ArcTan[-----]
          Sqrt[3]
Out[1]= -----
          Pi
```

```
In[2]:= % /. r -> 0
```

```
          1
Out[2]= -(-)
          2
```

```
In[3]:= Solve[%1 + 1/2 == 0.025, r]
```

```
Out[3]= {{r -> 0.0297866}}
```

Summary

- **Mathematica** can be useful for dealing with some tedious algebra or calculus.
- It is not really a substitute for **thinking**.
- Buy (or borrow) a book, or look for tutorials online.