A brief introduction to Mathematica

Karl W Broman
Department of Biostatistics
Johns Hopkins University

www.biostat.jhsph.edu/~kbroman

Why Mathematica?

C computational efficiency
Perl text/data manipulation
R interactive analyses

Mathematica symbolic algebra/calculus

Everything you could do back when you had just taken calculus, only accurately.

You might instead use Maple, but I have no experience with it.
Preliminaries

- Command-line version: type math
  (I use this, and copy-and-paste from a text file.)

- GUI (with mathematica “notebooks”): type mathematica

- To exit: type Quit

First stuff

In[1]:= 5^12
Out[1]= 244140625

In[2]:= %1 ^ (1/12)
Out[2]= 5

In[3]:= % + a
Out[3]= 5 + a

In[4]:= L = 3
Out[4]= 3

In[5]:= L
Out[5]= 3

In[6]:= L = 3;

In[7]:= 20 L
Out[7]= 60

In[8]:= 2 m + 3 m
Out[8]= 5 m

In[9]:= %7
Out[9]= 60

In[10]:= %8
Out[10]= 5 m
Buy a book, such as Abell & Braselton, *Mathematica by Example*, 3rd ed.

Use Google.

---

**Packages**

Sometimes, you need to load a separate package. I don’t recall ever needing this.

```
In[1]:= GramSchmidt[{{1,1,0}, {0,2,1}, {1,0,3}}]
Out[1]= GramSchmidt[{{1, 1, 0}, {0, 2, 1}, {1, 0, 3}}]

In[2]:= Remove[GramSchmidt]

In[3]:= << LinearAlgebra'Orthogonalization'

In[4]:= GramSchmidt[{{1,1,0}, {0,2,1}, {1,0,3}}]
```

```
            1 1 1 1 1
Out[4]= {{-------, -------, 0}, {-(-------), -------, -------},

1 1 2
> {-------, -(-------), Sqrt[-]}}
```
A bit of notation

[ ] Arguments to functions
{ } Lists
[[ ]] Subsetting lists

Numbers

\[
\begin{align*}
\text{In}[1] & \:= \ 5 \times 10 & \text{In}[5] & \:= \ 1/2 + 2/144 & \text{In}[7] & \:= \ \text{Sqrt}[27] \\
\text{Out}[1] & \:= \ 50 & \text{Out}[5] & \:= \ \text{--} & \text{Out}[7] & \:= \ 3 \ \text{Sqrt}[3] \\
\text{In}[2] & \:= \ 5 \ 10 & \text{In}[6] & \:= \ 1/2 + 2.0/144 & \text{In}[8] & \:= \ \text{Sqrt}[27.0] \\
\text{Out}[2] & \:= \ 50 & \text{Out}[6] & \:= \ 72 & \text{Out}[8] & \:= \ 5.19615 \\
\text{In}[3] & \:= \ a10 & \text{In}[7] & \:= \ \text{N[Sqrt}[27]] & \text{In}[9] & \:= \ \text{5.19615} \\
\text{Out}[3] & \:= \ a10 & \text{Out}[7] & \:= \ 0.513889 & \text{Out}[9] & \:= \ 5.19615 \\
\text{In}[4] & \:= \ a \ 10 & \text{Out}[4] & \:= \ a \ a \\
\end{align*}
\]
Constants

\[ \text{In[1]} := E \]
\[ \text{Out[1]} = E \]

\[ \text{In[2]} := \text{Pi} \]
\[ \text{Out[2]} = \text{Pi} \]

\[ \text{In[3]} := \text{N[E, 25]} \]
\[ \text{Out[3]} = 2.718281828459045235360287 \]

\[ \text{In[4]} := \text{N[Pi, 100]} \]
\[ \text{Out[4]} = 3.14159265358979323846264338327950288419716939937510582097494459230781640628620899866268034825342117068 \]

Algebra

\[ \text{Expand, Factor, Together, Apart, Simplify, FullSimplify} \]

\[ \text{In[1]} := \text{Expand[ (x + 2y + z)^2 ]} \]
\[ \text{Out[1]} = x^2 + 4 x y + 4 y^2 + 2 x z + 4 y z + z^2 \]

\[ \text{In[2]} := \text{Factor[ % ]} \]
\[ \text{Out[2]} = (x + 2 y + z)^2 \]

\[ \text{In[3]} := \text{Together[ 1/(1+2x) - 2/(2+3x) ]} \]
\[ \text{Out[3]} = \frac{x}{(1 + 2 x) (2 + 3 x)} \]

\[ \text{In[4]} := \text{Apart[ % ]} \]
\[ \text{Out[4]} = \frac{1}{1 + 2 x} - \frac{2}{2 + 3 x} \]
Solving equations

Solve, NRoots

In[1]:= f = x^3 - 3 x^2 - 17 x + 51;

In[2]:= soln = Solve[f == 0, x]
Out[2]= {{x -> 3}, {x -> -Sqrt[17]}, {x -> Sqrt[17]}}

In[3]:= f /. soln
Out[3]= {0, 0, 0}

In[4]:= NRoots[f == 0, x]

A silly example

Take \( V_g = \frac{a^2}{2} + \frac{d^2}{4}, \) \( V_e = \frac{V_g(1 - h^2)}{h^2} \), and \( a = 4d. \)

Supposing \( V_e = 1 \) and \( h^2 = 0.6 \), solve for \( d. \)

In[1]:= Vg = a^2 / 2 + d^2 / 4;
In[2]:= Ve = Vg (1-hsq)/hsq;
In[3]:= a = 4d;
In[4]:= hsq = 6/10;

In[5]:= Solve[Ve == 1, d]
Out[5]= {{d -> -Sqrt[--]}, {d -> Sqrt[--]}}
11 11

In[6]:= N[ % ]
Out[6]= {{d -> -0.426401}, {d -> 0.426401}}
Solving systems

Suppose we have $2p_1 + 2p_2 = 1$ and $p_1 = (1 - r)p_1 + p_2/2$.
Solve for $p_1$ and $p_2$.

\[
\text{In}[1]:= \text{eqn1} = 2 \, p_1 + 2 \, p_2 == 1; \\
\text{In}[2]:= \text{eqn2} = p_1 == (1-r) \, p_1 + p_2 \, / \, 2; \\
\text{In}[3]:= \text{Solve[ \{eqn1, eqn2\}, \{p_1, p_2\}]} \\
\text{Out}[3]= \{\{p_1 \rightarrow \frac{1}{2 (1 + 2 \, r)}, \, p_2 \rightarrow \frac{r}{1 + 2 \, r}\}\}
\]

A nonlinear example

Suppose $x^2 = 2y + 2$ and $x = y^2 + 1$.
Solve for $x$ and $y$.

\[
\text{In}[1]:= N[ \text{Solve[ \{x^2 == 2y + 2, x == y^2 + 1\}, \{x, y\}]} ] \\
\text{Out}[1]= \{\{x \rightarrow 2., \, y \rightarrow 1.\}, \{x \rightarrow 1.1304, \, y \rightarrow -0.361103\}, \\
> \{x \rightarrow -1.5652 - 1.04343 \, I, \, y \rightarrow -0.319448 + 1.63317 \, I\}, \\
> \{x \rightarrow -1.5652 + 1.04343 \, I, \, y \rightarrow -0.319448 - 1.63317 \, I\}\}
\]
Series

\textbf{In[1]}:= \text{Sum[ Exp[-\mu] \mu^n / Factorial[n], \{n, 0, Infinity\} ]} \\
\text{Out[1]}= 1

\textbf{In[2]}:= \text{Sum[ n Exp[-\mu] \mu^n / Factorial[n], \{n, 0, Infinity\} ]} \\
\text{Out[2]}= \mu

\textbf{In[3]}:= \text{Sum[ (n - \mu)^2 Exp[-\mu] \mu^n / Factorial[n], \{n, 0, Infinity\} ]} \\
\text{Out[3]}= \mu

\textbf{In[4]}:= \text{Sum[ p^k, \{k, 0, n\} ]} \\
\text{Out[4]}= \frac{1 + n}{-1 + p} \frac{-1 + p}{-1 + p}

\textbf{In[5]}:= \text{Sum[ p^k, \{k, 1, n\} ]} \\
\text{Out[5]}= \frac{n}{-1 + p} \frac{-1 + p}{-1 + p}

Limits

\textbf{In[1]}:= \text{Limit[ Sin[x]/x, x \to 0 ]} \\
\text{Out[1]}= 1

\textbf{In[2]}:= \text{Limit[ 1/x, x \to Infinity ]} \\
\text{Out[2]}= 0

\textbf{In[3]}:= \text{Limit[ 1/x, x\to0, Direction \to -1 ]} \\
\text{Out[3]}= \text{Infinity}

\textbf{In[4]}:= \text{Limit[ 1/x, x\to0, Direction \to 1 ]} \\
\text{Out[4]}= -\text{Infinity}
Integrals & derivatives

In[1]:= Integrate[ x^4 Cos[x], x ]
Out[1]= 4 x (-6 + x^2) Cos[x] + (24 - 12 x + x^2) Sin[x]

In[2]:= D[%, x]
Out[2]= 8 x Cos[x] + 4 (-6 + x^2) Cos[x] + (24 - 12 x + x^2) Cos[x] -
> 4 x (-6 + x^2) Sin[x] + (-24 x + 4 x^2) Sin[x]

In[3]:= Simplify[%]
In[4]:= Integrate[ Exp[x], {x, -1, 1} ]
E

In[5]:= Together[%]
Out[5]= -------
E

Another example

Consider $X_1, X_2, X_3 \sim \text{iid } N(\mu, \sigma^2)$.
Define $R = \frac{X(2) - X(1)}{X(3) - X(1)}$.

One can show that the density of $R$ is $f(r) = \frac{3\sqrt{3}}{2\pi} \cdot \frac{1}{r^2 + r(1-r) + (1-r)^2}$.

Find the cdf.

In[1]:= Integrate[ 3 Sqrt[3]/(2 Pi) / (r^2 + r(1-r) + (1-r)^2), r]}
Out[1]= \frac{-1 + 2 r}{3 \text{ArcTan}[-]} \frac{\text{Sqrt}[3]}{\text{Pi}}

In[2]:= % /. r -> 0
Out[2]= \frac{-(-)}{2}

In[3]:= Solve[%1 + 1/2 == 0.025, r]
Out[3]= \{r \rightarrow 0.0297866\}
Summary

- Mathematica can be useful for dealing with some tedious algebra or calculus.
- It is not really a substitute for thinking.
- Buy (or borrow) a book, or look for tutorials online.