Role of probability: expectation vs result

Example: I have a pocket full of candy. If the candy is half chocolate and half licorice and there are 5 pieces of each,

what are the chances of picking one of each type if I only pick two pieces? (probability)
Example: I have a pocket full of candy. If the candy is half chocolate and half licorice and there are 5 pieces of each,

If I pick 5 pieces and they are all licorice, should I trust that the chocolates are still there? (statistics)
“number sense”

• With small numbers, humans do fine
  • Family with 6 boys
  • All Brazil nuts at the top
• In fact, maybe we’re too good at seeing connections
  • Superstitions
  • Horoscopes
• With big numbers, human brains fall flat
  • Lottery
  • Microarray & sequencing experiments
Bayes’ theorem

balls in an urn: colors are blue or black, can be striped or solid

event A = ball I choose is blue

event B = ball I choose has stripes

\[ p(A) = \frac{23}{50} \]
\[ p(B) = \frac{28}{50} \]
Bayes’ theorem

event A = ball I choose is blue \hspace{1cm} p(A) = 23/50
event B = ball I choose has stripes \hspace{1cm} p(B) = 28/50

\[ p(A,B) = \text{probability that both A and B occur simultaneously} \]
\[ p(A,B) = p(B,A) = 13/50 \]

I draw a striped ball. What is the probability that it is blue?

\[ p(A|B) = \text{probability that A will occur, given that B has already happened} \]
\[ p(A|B) = \frac{p(A,B)}{p(B)} \]
\[ p(B|A) = \frac{p(A,B)}{p(A)} \]
\[ p(A,B) = p(A|B)p(B) \]
\[ = p(B|A)p(A) \]
Bayes’ theorem

event A = ball I choose is blue \quad \text{p(A)} = 23/50

event B = ball I choose has stripes \quad \text{p(B)} = 28/50

I draw a striped ball. What is the probability that it is blue?

\text{p(A|B)} = \text{probability that A will occur, given that B has already happened}

\text{p(A|B)} = \frac{\text{p(A,B)}}{\text{p(B)}}

\text{p(B|A)} = \frac{\text{p(A,B)}}{\text{p(A)}}

\text{p(A|B)p(B)} = \text{p(B|A)p(A)}

\text{p(A|B)} = \frac{\frac{\text{p(B|A)}}{\text{p(B)}} \times \text{p(A)}}{\text{p(A)}}

\text{p(A|B)} = \frac{\text{p(A,B)}}{\text{p(B)}} = \frac{13/50}{28/50} = 13/28
probability of a complex event

\[ p(X) = p(X|A) \ p(A) + p(X|B) \ p(B) + \ldots \]

equation of the sun rises in Baltimore before 7AM?

\[ p(S) = p(S|summer) \ p(summer) + p(S|fall) \ p(fall) + p(S|winter) \ p(winter) + p(S|spring) \ p(spring) \]