Syphilis – is it still a problem in East Baltimore?

The Binomial Equation in Practice
Syphilis in East Baltimore

- Syphilis has been reported as a major problem in East Baltimore
- Steps have been taken to reduce the rate
- In a (hypothetical) study of 100 randomly selected people in East Baltimore, 2 syphilis cases were identified
Syphilis in East Baltimore

- The overall syphilis rate for the entire country in 1999 was about 1.25 per 10,000.
- Using the Binomial Equation, we can decide whether the rate of 2 per 100 people in East Baltimore is excessive.
- *What is the Binomial Equation?*
What is the Binomial Equation?

- We have an unfair coin: the probability of getting a tail is 0.6

- What is the probability that one out of four students will get a tail?
What is the Binomial Equation?

<table>
<thead>
<tr>
<th>Benilton</th>
<th>Qing</th>
<th>Weiwei</th>
<th>Yenyi</th>
</tr>
</thead>
<tbody>
<tr>
<td>H or T</td>
<td>H or T</td>
<td>H or T</td>
<td>H or T</td>
</tr>
<tr>
<td>0.4 or 0.6</td>
<td>0.4 or 0.6</td>
<td>0.4 or 0.6</td>
<td>0.4 or 0.6</td>
</tr>
</tbody>
</table>

\[0.6 \times 0.4 \times 0.4 \times 0.4 = 0.0384\]
What is the Binomial Equation?

- But suppose Qing had gotten tails instead of Benilton

- The probability still equals

$$0.6 \times 0.4 \ (3) = 0.0384$$
Building the Binomial Equation

- Probability of getting 1 tail overall is

\[
\binom{4}{1} \times 0.6^1 \times 0.4^{4-1}
\]
Building the Binomial Equation

- How many ways can 0 people out of 4 get tails?
- How many ways can 1 person out of 4 get tails?
- How many ways can 2 people out of 4 get tails?
Building the Binomial Equation

- Formula: “4 choose x”

\[
\binom{4}{x} = \frac{4!}{x!(4-x)!}
\]
Building the Binomial Equation

\[
\binom{4}{0} = \frac{4!}{0!(4-0)!} = \frac{4 \times 3 \times 2 \times 1}{1 \times (4 \times 3 \times 2 \times 1)} = 1
\]

\[
\binom{4}{1} = \frac{4!}{1!(4-1)!} = \frac{4 \times 3 \times 2 \times 1}{1 \times (3 \times 2 \times 1)} = 4
\]

\[
\binom{4}{2} = \frac{4!}{2!(4-2)!} = \frac{4 \times 3 \times 2 \times 1}{2 \times 1 \times (2 \times 1)} = \frac{12}{2} = 6
\]
The Binomial Equation

\[
\binom{N}{x} p^x (1-p)^{(N-x)} \times 0.6^{(1)} \times 0.4^{(4-1)}
\]

where
- \(N=4\)
- \(P=0.6\)
- \(x=1\)
Using the Binomial Equation

- What is the probability that *exactly* 2 people out of 100 will have syphilis if the rate in East Baltimore is the same as the rate in the U.S. (1.25 per 10,000)?

\[
\text{Prob}(X=2) = \binom{N}{x} p^x (1-p)^{N-x}
\]

where

- \(N=100\)
- \(p=0.000125\)
- \(x=2\)

\[
= \binom{100}{2} 0.000125^2 (1-0.000125)^{100-2} = 0.000076
\]
Using the Binomial Equation

- What is the probability that at least 2 people out of 100 will have syphilis?

\[
\text{Prob}(X \geq 2) = 1 - \text{Prob}(X < 2) = 1 - \text{Prob}(X \leq 1) = 1 - \text{Prob}(X = 0) - \text{Prob}(X = 1)
\]
Using the Binomial Equation
Using the Binomial Equation

1-\text{Prob}(X=0)-\text{Prob}(X=1):

\text{Prob}(X=0) = 0.9876
\text{Prob}(X=1) = 0.0123

\text{Prob}(X\geq 2) = 1-0.9876-0.0123 = 0.001

There is only 1 chance in 1000 of seeing 2 or more people with syphilis out of 100 people, if the rate in East Baltimore is the same as in the U.S.
Conclusion

- The *observed* data in Baltimore is *extremely unlikely* if the Baltimore syphilis rate is the same as the U.S. rate.

- Therefore we conclude that the Baltimore rate is higher than the U.S. rate.
Assumptions

- Everyone in East Baltimore has the same rate of syphilis ($p=\text{constant}$)

- Which also means no one in our sample is related to one another:
  
  If Jon has syphilis, then the probability that his wife Janice has syphilis should increase