On Likelihood Inference under Nonidentifiability with Applications to Biomedical Studies

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Conventional asymptotic theory of likelihood inference relies critically on a set of regularity conditions. When one or more such conditions are violated, the likelihood ratio test (LRT) statistic generally does not converge to a chi-square distribution. In this talk, we discuss the asymptotic behavior of the LRT and its modifications when one such condition, namely, identifiability, fails to hold. Two distinct classes of problems are of particular interest. In the first class, the null hypothesis is specified via the parameter of interest while a nuisance parameter is not identifiable under the null. This class of problems has been studied extensively in the last few decades (e.g., Davies 1977, 1987), and we will briefly review their results. In the second class, the null hypothesis can be specified equivalently via each one of the two parameters, and under either specification, the other parameter is not identifiable. Examples include testing homogeneity in admixture models, testing linearity versus a specific nonlinear trend in regression models, and testing the existence of a change point. We show that in this situation, the LRT statistic converges to the supremum of a squared Gaussian process. We also present modifications of the LRT that lead to a simple chi-square asymptotic approximation under the null, for instance, through the concept of modified likelihood (Chen *et al.* 2001). These approaches are compared with the LRT through statistical power and are illustrated in a genetic linkage study of schizophrenia that is subject to genetic heterogeneity.