BST 140.778 Assignment 2

February 2, 2005

Fine print Please feel free to give each other small hints, but otherwise students must complete assignments individually. Assignments can be hand written, but note that sloppily prepared work will be returned ungraded.

1. A glm with a canonical link and $\phi_i = \phi$ has sufficient statistic $S = X^t y$ (using the notation from class). Assume that T is a goodness of fit statistic, such as the deviance or Pearson statistic. Assume t_0 is the observed value for T. If the slope parameters, β, are known, then a P-value for model fit is given by $P(T \geq t_0; \beta)$. Explain why $P(T \geq t_0 | S)$ is the uniform minimum variance unbiased estimator of P.

2. Suppose that y_i is Poisson with $g(\mu_i) = \alpha + \beta x_i$ where g is the link function and $x_i = 1$ for $i = 1, \ldots, n_a$ and $x_i = 0$ for $i = n_a + 1, \ldots, n_a + n_b$. That is, x_i, is a treatment indicator for two groups, A and B. Show that, regardless of the link function, the fitted means equal the two sample means.

3. Consider the class of *binary* glms where the link function satisfies $g\{\mu(x)\} = \Phi^{-1}\{\mu(x)\} = \alpha + \beta x$ where $\Phi(\cdot)$ is a distribution function and $\mu(x)$ is the Bernoulli mean. Let ϕ be the (assumed continuous) associated density. Show that the x at which $\mu(x) = .5$ is $x = -\alpha/\beta$. Further show that the rate of change of $\mu(x)$ at this point is $\beta\phi(0)$. Illustrate that this is $.25\beta$ for the logit link and $\beta/\sqrt{2\pi}$ for the probit link.

4. McCullaugh and Nelder problem 4.17.

5. Give the title and a list of key references for your second year oral exam paper.