BST 140.753 Assignment 3

February 16, 2005

Fine print Please feel free to give each other small hints, but otherwise students must complete assignments individually. Assignments can be hand written, but note that sloppily prepared work will be returned ungraded.

1. The table below shows the temperature (Temp in Fahrenheit) and presence (1) or absence of O-ring distress (OD) at the time of flight for the 23 flights before the 1986 Challenger mission disaster.

Temp	OD	Temp	OD	Temp	OD	Temp	OD	Temp	OD
66	0	70	1	69	0	68	0	67	0
72	0	73	0	70	0	57	1	63	1
70	1	78	0	67	0	53	1	67	0
75	0	70	0	81	0	76	0	79	0
75	1	76	0	58	1				

   a. Use logistic regression to model the effect of temperature on the probability of thermal distress. Interpret the results. Plot a figure of the fitted model.

   b. Estimate the probability of thermal distress at 31 degrees, which was the temperature at the time of the Challenger flight.

   c. Construct a profile likelihood for the effect of temperature on the odds of thermal distress, interpret.

   d. Check model fit by comparing this model to a more complex model.

2. Suppose that Y has a binomial(n, π) distribution. Consider the model $\text{logit}(\pi) = \alpha$ and consider testing $H_0 : \alpha = 0$. Let $\hat{\pi} = y/n$.

a. Argue that the asymptotic variance of $\hat{\alpha}$ is $\{n\pi(1-\pi)\}^{-1}$. Consider the test statistic $(\hat{\alpha}/SE)^2$, where SE is the standard error for the numerator. Make a plot of the test statistic using the estimated standard error, i.e. setting $\pi = \hat{\pi}$ is SE by the value of $0 < \hat{\pi} < 1$. On the same plot, plot the test statistic where the standard error uses the null value of π (which is .5) versus $\hat{\pi}$. Intepret the results.

b. Create the same plots for the test statistic $\{(\hat{\pi}-.5)/SE\}^2$ where SE is now the standard error for $\hat{\pi}$. Intepret the results.

c. Suppose that $y = 0$ or $y = n$. Show that the Wald test in part a. cannot reject $H_0 : \pi = \pi_0$ for any $0 < \pi_0 < 1$ while the Wald test in b. rejects for every such π_0.

3. Consider a table where respondents were asked their approval/disapproval of the prime mister on two occasions.

	First Survey	
Second Survey	Approve	Disapprove
Approve	794	150
Disapprove	86	570

Table 1: Approval ratings

Let π_{ij} be the true multinomial cell probabilities ($\sum_{ij} \pi_{ij} = 1$) and n_{ij} represent the cell counts for a 2×2 table. Consider modeling the marginal probabilities that a subject approves of the prime minister's performance

$$\text{logit}(\pi_{1+}) = \alpha,$$

and

$$\text{logit}(\pi_{+1}) = \alpha + \beta.$$

a. Show that the ML estimate of β is $\hat{\beta} = \log(p_{+1}p_{2+}/p_{1+}p_{+2})$.

b. Use the delta method to show that the asymptotic variance of $\sqrt{n}(\hat{\beta} - \beta)$ is

$$(\pi_{1+}\pi_{2+})^{-1} + (\pi_{+1}\pi_{+2})^{-1} - 2(\pi_{11}\pi_{22} - \pi_{12}\pi_{21})/(\pi_{1+}\pi_{2+}\pi_{+1}\pi_{+2})$$

c. Use the previous results to analyze marginal homogeneity of the prime minister's approval rating responses.

4. Give the title, some references and a brief outline of your second year oral exam paper.