Advanced Theory

Survival Analysis 2005

Problem for March 1, 2005

$$\chi^2_{MH} = (\sum a_i - \sum \hat{A}_i)^2 / \sum V_i$$

$$\chi^2_0 = (\sum a_i - \sum \hat{A}_i)^2 / \sum \hat{A}_i + (\sum c_i - \sum \hat{C}_i)^2 / \sum \hat{C}_i$$

Prove $\chi^2_{MH} > \chi^2_0$.

Let's assume the simple case when we have just 1 event in the at-risk group at any time period. Hence, $a_i + c_i = 1$. Note, that this implies that $b_i + d_i = n_i - 1$ since $a_i + b_i + c_i + d_i = n_i$. Hence,

$$\chi^2_{MH} = \left(\sum a_i - \frac{(a_i + b_i)(a_i + c_i)}{n_i}\right)^2 / \sum \frac{(a_i + b_i)(c_i + d_i)}{n_i^2}$$

$$= \left(\sum a_i - \frac{(a_i + b_i)}{n_i}\right)^2 / \sum \frac{(a_i + b_i)(c_i + d_1)}{n_i^2}$$

$$\chi^2 = \left(\sum a_i - \frac{(a_i + b_i)(a_i + c_i)}{n_i}\right)^2 / \sum \frac{(a_i + b_i)(a_i + c_i)}{n_i} + \left(\sum c_i - \frac{(c_i + d_i)(a_i + c_i)}{n_i}\right)^2 / \sum \frac{(c_i + d_i)(a_i + c_i)}{n_i}$$

$$\chi^2 = \left(\sum a_i - \frac{(a_i+b_i)}{n_i}\right)^2 / \sum \frac{(a_i+b_i)}{n_i} + \left(\sum c_i - \frac{(c_i+d_i)}{n_i}\right)^2 / \sum \frac{(c_i+d_i)}{n_i}$$

$$= \left(\sum a_i - \frac{(a_i+b_i)}{n_i}\right)^2 / \sum \frac{(a_i+b_i)}{n_i} + \left(\sum (1-a_i) - \frac{(1-a_i+n_i-1-b_i)}{n_i}\right)^2 / \sum \frac{(c_i+d_i)}{n_i}$$

$$= \left(\sum a_i - \frac{(a_i+b_i)}{n_i}\right)^2 / \sum \frac{(a_i+b_i)}{n_i} + \left(\sum a_i - \frac{(a_i+b_i)}{n_i}\right)^2 / \sum \frac{(c_i+d_i)}{n_i}$$

Note that $(a_i+b_i)/n_i \leq 1$ and $(c_i+d_i)/n_i \leq 1$, and $(a_i+b_i+c_i+d_i)/n_i = 1$ so

$$\leq \frac{1}{2}\left[\left(\sum a_i - \frac{(a_i+b_i)}{n_i}\right)^2 / \sum \frac{(a_i+b_i)(c_i+d_i)}{n_i^2} + \left(\sum a_i - \frac{(a_i+b_i)}{n_i}\right)^2 / \sum \frac{(a_i+b_i)(c_i+d_i)}{n_i^2}\right]$$
$$= \chi_{MH}$$