Advanced Theory

Survival Analysis 2005

Problem for February 1, 2004

1) Prove:

$$\text{area} = E(T) = \int_0^\infty S(t)dt$$

a) when S(t) $= \exp(-\lambda t)$, then T has a negative exponential distribution (sometimes this is just called an exponential distribution).

$$\int_0^\infty \exp(-\lambda t)dt$$

$$= -\frac{1}{\lambda}\exp(-\lambda t) \mid_0^\infty$$

$$= 0 - \left(-\frac{1}{\lambda}\right)$$

$$= \frac{1}{\lambda}$$

which is the mean of a variable with an exponential distribution.

b) In general for non-negative random T:

$$E[T] = \int_0^\infty tf(t)dt$$

$$= \int_0^\infty \int_0^\infty I(0 \le u \le t)du\ f(t)\ dt$$

$$= \int_0^\infty \int_0^\infty I(0 \le u \le t)f(t)\ \ dt\ du$$

$$= \int_0^\infty \int_u^\infty f(t)\ dt\ du$$

$$= \int_0^\infty S(u)\ du$$