1. Let \( X_1, \ldots, X_n \) be i.i.d. \( \text{Binomial}(k, p) \) random variables, where \( k \) is an unknown integer and \( p \in [0, 1] \).

   • Derive a method of moment estimator for \( k \) and \( p \). Comment on the appropriateness of these estimators.
   
   • Show how you would compute the maximum likelihood estimator for \( k \) and \( p \)? Fix \( k \) and find MLE for \( p \) in terms of \( k \), say \( \hat{p}(k) \). What is \( \hat{p}(k) \)? Then plug \( \hat{p}(k) \) into the likelihood and maximize with respect to \( k \). What values of \( k \) should you maximize the likelihood over?

2. Suppose \( W \) and \( X \) have a known joint density denoted by \( q \). Suppose that the conditional density of \( Y \) given \( W \) and \( X \) is normal with mean \( \alpha W + \beta X \) and variance 1, \( \alpha \in \mathbb{R}, \beta \in \mathbb{R} \). Suppose \((W_1, X_1, Y_1), \ldots, (W_n, X_n, Y_n)\) are i.i.d. with joint density as described.

   • Derive the MLE for \( \alpha, \beta \). What is the limiting distribution of the MLE for \( \alpha \)?
   
   • Suppose \( \beta \) is known. Derive the MLE for \( \alpha \). What is the limiting distribution of the MLE for \( \alpha \)?
   
   • When will the limiting distributions in the previous two parts be the same.

3. Let \( X_1, \ldots, X_n \) be iid \( U(\theta, 2\theta) \) where \( \theta > 0 \).

   • Find a method of moments estimator for \( \theta \).
   
   • Find the MLE, \( \hat{\theta}_n \).