1. Let $Y_1, \ldots, Y_n$ be independent $Bernoulli(p_i)$, where

$$p_i = \frac{\exp(\alpha + \beta x_i)}{1 + \exp(\alpha + \beta x_i)},$$

$x_i$’s can be considered fixed and $-\infty < \alpha, \beta < \infty$. Derive a complete, sufficient statistic for $(\alpha, \beta)$.

2. Suppose that $X \sim Gamma(\alpha, \beta)$. Show that the Gamma distribution belongs to a two-parameter exponential family. Find its natural parametrization. Find the mean of $t_1(X)$ and $t_2(X)$ and the covariance between $t_1(X)$ and $t_2(X)$. Suppose $X_1, \ldots, X_n$ are i.i.d. $Gamma(\alpha, \beta)$. Find a complete, sufficient statistic for $(\alpha, \beta)$.

3. Explain why the negative binomial distribution with unknown parameters $r$ and $p$ is not of exponential family form.

4. YS: 5.3

5. CB: 6.9

6. Suppose $X_1, \ldots, X_n$ are i.i.d. $Uniform(\theta, \theta + 1)$. Find a MMS for $\theta$. Is this statistic complete?

7. CB 7.49

8. CB 7.44

9. CB 7.57

10. CB 7.58