

Homework Assignments #1

1. Suppose that there is a Beta(4,4) prior distribution on the probability θ that a coin will yield a “head” when spun in a specified manner. The coin is independently spun 10 times, and “heads” appear fewer than 3 times. You are not told how many heads were seen, only that the number is less than 3 times. Calculate your exact posterior, posterior mean, variance, 95 % posterior intervals.

2. Let y be the number of heads in n spins of a coin, whose probability of heads is θ .

(a) If your prior distribution for θ is uniform on the range $[0, 1]$, derive your prior predictive distribution for y :

$$P(y = k) = \int_0^1 Pr(y = k | \theta) d\theta$$

(b) Suppose you assign a Beta(α, β) prior distribution for θ , and then you observe y heads out of n spins. Show algebraically that the posterior mean of θ always lies between the prior mean $\frac{\alpha}{\alpha+\beta}$, and the observed relative frequency of heads $\frac{y}{n}$.