Table 1: Observed effects of special preparation on SAT-V scores in eight randomized experiments. Rubin (1981)

<table>
<thead>
<tr>
<th>School</th>
<th>Estimated treatment effect, $y_j$</th>
<th>Standard error of effect estimate, $\sigma_j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>28.39</td>
<td>14.9</td>
</tr>
<tr>
<td>B</td>
<td>7.94</td>
<td>10.2</td>
</tr>
<tr>
<td>C</td>
<td>-2.75</td>
<td>16.3</td>
</tr>
<tr>
<td>D</td>
<td>6.82</td>
<td>11.0</td>
</tr>
<tr>
<td>E</td>
<td>-0.64</td>
<td>9.4</td>
</tr>
<tr>
<td>F</td>
<td>0.63</td>
<td>11.4</td>
</tr>
<tr>
<td>G</td>
<td>18.01</td>
<td>10.4</td>
</tr>
<tr>
<td>H</td>
<td>12.16</td>
<td>17.6</td>
</tr>
</tbody>
</table>

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1. For each of the following densities provide a conjugate prior distribution for the unknown parameter(s), if one exists:
   a) $y \sim \text{Binomial}(n, \theta)$, $n$ known  
   b) $y \sim \text{Neg-Bin}(r, \theta)$, $r$ known  
   c) $y \sim \text{Gamma}(\alpha, \beta)$, $\alpha$ known  
   d) $y \sim \text{Gamma}(\alpha, \beta)$, $\beta$ known

2. Suppose your prior distribution for $\theta$, the proportion of Californians who support the death penalty, is Beta with mean 0.6 and standard deviation 0.3.
   (a) Determine the parameters $\alpha$ and $\beta$ for your prior distribution. Sketch the prior density function;  
   (b) a random sample of 1000 Californians is taken, and 65% support the death penalty. What are the posterior mean and variance for $\theta$? Draw the posterior density function.

3. Data analysis of the SAT coaching example.
   (a) Let $y_j$ be the estimated coaching effects and let $\sigma_j^2$ the corresponding standard error for school $j$ as summarized in the Table. We assume $y_j \sim N(\theta_j, \sigma_j^2)$, $j = 1, \ldots, J$.
   (b) Implement a classical random-effects analysis of variance of the SAT coaching data, and estimate $\theta_j$ for $j = 1, \ldots, 8$;  
   (c) elicit a prior distribution for $\mu$ and $\tau^2$ and perform a fully Bayesian analysis of the following hierarchical model

\[
y_j \sim N(\theta_j, \sigma_j^2) \\
\theta_j \sim N(\mu, \tau^2);
\]

you can use direct simulation or Gibbs sampling

(d) use the posterior simulations to estimate (i) for each school $j$, the probability that its coaching program is the best of the eight; and (ii) for each pair of schools, $j$ and $k$, the probability that the coaching program in school $j$ is better than in school $k$;

(e) repeat (a) but for $\tau = \infty$ (that is, separately estimation for the eight schools). In this case, the probabilities (ii) can be computed analytically;
(f) discuss how the answers in (d) and (e) differ;

(g) pick one school and investigate the sensitivity of the posterior probability that its coaching program is the best of the eight schools under 2 alternative prior specifications for \( r^2 \).