Table 1: *Observed effects of special preparation on SAT-V scores in eight randomized experiments. Rubin (1981)*

School	Estimated treatment effect, y_j	Standard error of effect estimate, σ_j
A	28.39	14.9
B	7.94	10.2
C	-2.75	16.3
D	6.82	11.0
E	-0.64	9.4
F	0.63	11.4
G	18.01	10.4
H	12.16	17.6

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1. For each of the following densities provide a conjugate prior distribution for the unknown parameter(s), if one exists:
   a) $y \sim$ Binomial(n, θ), n known
   b) $y \sim$ Neg-Bin(r, θ), r known
   c) $y \sim$ Gamma(α, β), α known
   d) $y \sim$ Gamma(α, β), β known

2. Suppose your prior distribution for θ, the proportion of Californians who support the death penalty, is Beta with mean 0.6 and standard deviation 0.3.
   (a) Determine the parameters α and β for your prior distribution. Sketch the prior density frunction;
   (b) a random sample of 1000 Calfornians is taken, and 65% support the death penalty. What are the posterior mean and variance for θ? Draw the posterior density function.

3. Data analysis of the SAT coaching example.
   (a) Let y_j be the estimated coaching effects and let σ_j^2 the corresponding standard error for school j as summarized in the Table. We assume $y_j \sim N(\theta_j, \sigma_j^2)$, $j = 1, \ldots, J$.
   (b) implement a classical random-effects analysis of variance of the SAT coaching data, and estimate θ_j for $j = 1, \ldots, 8$;
   (c) elicit a prior distribution for μ and τ^2 and perform a fully Bayesian analysis of the following hierarchical model

$$\begin{aligned} y_j &\sim N(\theta_j, \sigma_j^2) \\ \theta_j &\sim N(\mu, \tau^2); \end{aligned}$$

   you can use direct simulation or Gibbs sampling
   (d) use the posterior simulations to estimate (i) for each school j, the probability that its coaching program is the best of the eight; and (ii) for each pair of schools, j and k, the probability that the coaching program in school j in better than in school k;
   (e) repeat (a) but for $\tau = \infty$ (that is, separately estimation for the eight schools). In this case, the probabilities (ii) can be computed analytically;

(f) discuss how the answers in (d) and (e) differ;

(g) pick one school and investigate the sensitivity of the posterior probability that its coaching program is the best of the eight schools under 2 alternative prior specifications for τ^2.