Bayesian Methods

LABORATORY

Lesson 3: Feb 07 2002

Software: **BUGS and R**

Multi-parameter models: Normal and Multinomial

MULTI-PARAMETER *models*

Two examples:

1. The univariate Normal model with unknown mean μ and variance σ^2
2. The Multinomial model

1. Cancer survival times

Congdon's book, pag. 21, Example 2.4
Program 2.4 Carcinoma Survival

- Aitchinson and Dunsmore (1975) present data on survival times y in weeks after a combination of radiotherapy and surgery applied to a particular carcinoma.
- One question of interest is the length of survival expected for a new patient with this carcinoma and assigned to this type of treatment.
- Because of the skewed form of the data we apply a log transformation and assume that y is log-normal.

normal example continues ...

- Another question of interest is about the probability that a new patient has a survival time exceeding 150 (log=5.01)
- Finally, it may be of interest to examine age effects. For instance, we can dichotomize the ages of the patients into those below and above 50.
- Suppose we, first, assume a non informative prior,
- but secondarily we are willing to specify a prior belief that the expected mean length of survival is 30 days (log=3.4) with a 'size of 10', and that the variance is 2 with a degree of 'strength of 10'.

Inference on normal parameters: mean and variance unknown

A) Independent non informative prior $p(\mu)\, p(\sigma^2)$

with R ...

- Non informative prior: $p(\mu, \sigma^2) \propto 1/\sigma^2$
- posterior calculations by **marginal and conditional simulation**: $p(\mu, \sigma^2|y) = p(\mu|\sigma^2, y)p(\sigma^2|y)$
- Marginal posterior distribution for the variance:

$$p(\sigma^2|y) \sim Inv -_{\chi^2}\, (n-1, s^2)$$

- Conditional posterior distribution for the mean:

$$p(\mu|\sigma^2, y) = N(\bar{y}, \sigma^2/n)$$

- Posterior predictive distribution for y_{n+1}:

$$p(y_{n+1}|\mathbf{y}) = \int p(y_{n+1}|\mu, \sigma^2, \mathbf{y})\, p(\mu, \sigma^2|\mathbf{y})\, d\mu d\sigma^2$$

(i) conditional simulation given the drawn (μ, σ^2) a posteriori:

$$p(y_{n+1}|\mu, \sigma^2, \mathbf{y}) = Norm(y_{n+1}|\mu, \sigma^2)$$

(ii) or, marginal simulation:

$$p(y_{n+1}|\mathbf{y}) = t_{n-1}(\bar{y}, (1+\tfrac{1}{n})^{1/2}s)$$

- Probability of ($y_{n+1} > 5.01 (= log150)$)
(i) simulation of a $f(y_{n+1})$: $I_{[y_{n+1}>5.01]}(y_{n+1})$
(ii) or, probability computation:

$$1 - F_{t_{n-1}}((5.01 - \bar{y})/(1+\tfrac{1}{n}s)$$

R code $\longrightarrow$ sigma.post, mu.cond.post, y.new, g.150

```
norm2.r_function(m) {
 ...
y_log(surv)
s2_1/(n-1)*sum((y-mean(y))^2)
chi_rchisq(m,n-1)
sigma.post_(n-1)*s2/chi
y.bar_mean(y)
for(i in 1:m) {
mu.post.cond_rnorm(1,y.bar,sqrt(sigma.post[i]/n))
y.new_c(y.new,rnorm(1,mu.post.cond[i],sqrt(sigma.pc
g.150_c(g.150,as.numeric(y.new[i]>5.01))}
#ti_rt(10000,19)
#y.new_sqrt(1+1/n)*sqrt(s2)*ti+y.bar
#g.150_1-pt((log(150)-y.bar)/(sqrt(1+1/n)*sqrt(s2))
```


Independent non informative prior $p(\mu)\, p(\sigma^2)$

with Bugs ...

BUGS *requires that a* ***full (joint) probability model*** *is defined, and hence* **forces all priors to be proper** *(specifically the priors on 'founder nodes' of the DAG graph) Then 'just proper' priors are usually specified. E. g., in our example:*

- non informative prior for the mean:
  $N(0, \tau = 0.0001)$ *locally uniform over* ± 200
- non informative prior for the precision τ:
  $G(0.0001, 0.0001) \quad \sim 1/\tau$
- posterior calculations by Gibbs sampling

1. Cancers in women

Congdon's book, pag. 39, Example 2.16
Program 2.16 Female Cancers

- In 1995, breast cancer in the UK accounted for 18% female cancer deaths.
- Suppose we were interested in the credible interval of this rate, in a situation where we have k=11 cancer types.
- We assume a non informative $Dirichlet(\mathbf{c})$, with $c_i = 1,\ i = 1, \ldots, k$. *Then, the 1995 female cancer data, for which the total deaths X=76680, will tend to 'swamp' the prior.*

multinomial example continues ...

- Apart from the *multinomial procedure*, the other option is to estimate the multinomial parameters by taking each of the k=11 outcomes as independent Poissons.
- We may assume non informative Gamma priors on each Poisson mean, and then calculate, for each draw from the posterior forms of these 11 distributions, the estimates $\theta_i = \mu_i / \sum_i \mu_i$.

See at the web course page mnomial.r and mnomial.b for the codes in R and BUGS.

B) Interdependent priors: $p(\mu|\sigma)\,p(\sigma^2)$ with R...

- conjugate prior: $N - Inv_{\chi^2}(\mu_0, \sigma_0^2/m_0\,;\,\nu_0, \sigma_0^2)$

$$p(\mu|\sigma^2)p(\sigma^2) = N(\mu_0, \sigma^2/m_0)\ Inv_{\chi^2}(\nu_0, \sigma_0^2)$$

- posterior calculations from the posterior $N - Inv_{\chi^2}(\mu_n, \sigma_n^2/m_n\,;\,\nu_n, \sigma_n^2)$ by **marginal and conditional simulation**: $p(\mu, \sigma|y) = p(\mu|\sigma, y)p(\sigma|y)$
- Marginal posterior distribution for the variance:

$$p(\sigma^2|y) \sim Inv_{\chi^2}(\nu_n, \sigma_n^2)$$

- Conditional posterior distribution for the mean:

$$p(\mu|\sigma, y) = N(\mu_n, \sigma^2/m_n)$$

Interdependent informative prior $p(\mu|\sigma)\,p(\sigma^2)$ with Bugs ...

- Prior for the mean: $N(\mu_0, t_0 * \tau)$
  In our example on survival time: $\mu_0 = 3.4$ (=log 30 days), prior sample size $t_0 = 10$
- Prior for the precision: $G(\nu_0/2, \nu_0\,\sigma_0^2/2)$
  In our example: $\sigma_0^2 = 2$, prior $d.f. = 10$
- posterior calculations by Gibbs sampling

See at the web course page normal2.r, normal2.dat and normal2.b for the codes in R and BUGS.