Our 3-level random-intercept model:

$$\log\left(\frac{p(y_{ijk}=1)}{1-p(y_{ijk}=1)}\right)=\eta_{ijk}$$

$$\eta_{ijk}=\beta_0+\beta_1 kid2p_{ijk}+\beta_2 indNoSpa_{jk}+\ldots+\beta_{10} pcInd81_k+U_{jk}+U_k$$

The above model can be decomposed as:

First level (Child):

$$\text{logit}\,(\,P\,(y_{ijk}=1)\,)\;=\eta_{jk}+\beta_1\ \text{kid2p}_{ijk}$$

Second level (Mother):

$$\begin{aligned}\eta_{jk} = \quad \eta_k + \quad & \beta_2\ \text{indNoSpa}_{jk}+\beta_3\ \text{indSpa}_{jk}+\beta_4\ \text{momEdPri}_{jk}+\\ & \beta_5\ \text{momEdSec}_{jk}+\beta_6\ \text{husEdPri}_{jk}+\beta_7\ \text{husEdSec}_{jk}+\\ & \beta_8\ \text{husEdDK}_{jk}\ +U_{jk}\end{aligned}$$

$$U_{jk}\sim \text{Normal}\,(\,0,\,\tau^2{}_1)$$

Third level (Cluster):

$$\eta_k = \quad \beta_0+\beta_9\ \text{rural}_k+\beta_9\ \text{pcInd81}_k\ +U_k$$

$$U_k\sim \text{Normal}\,(\,0,\,\tau^2{}_2)$$

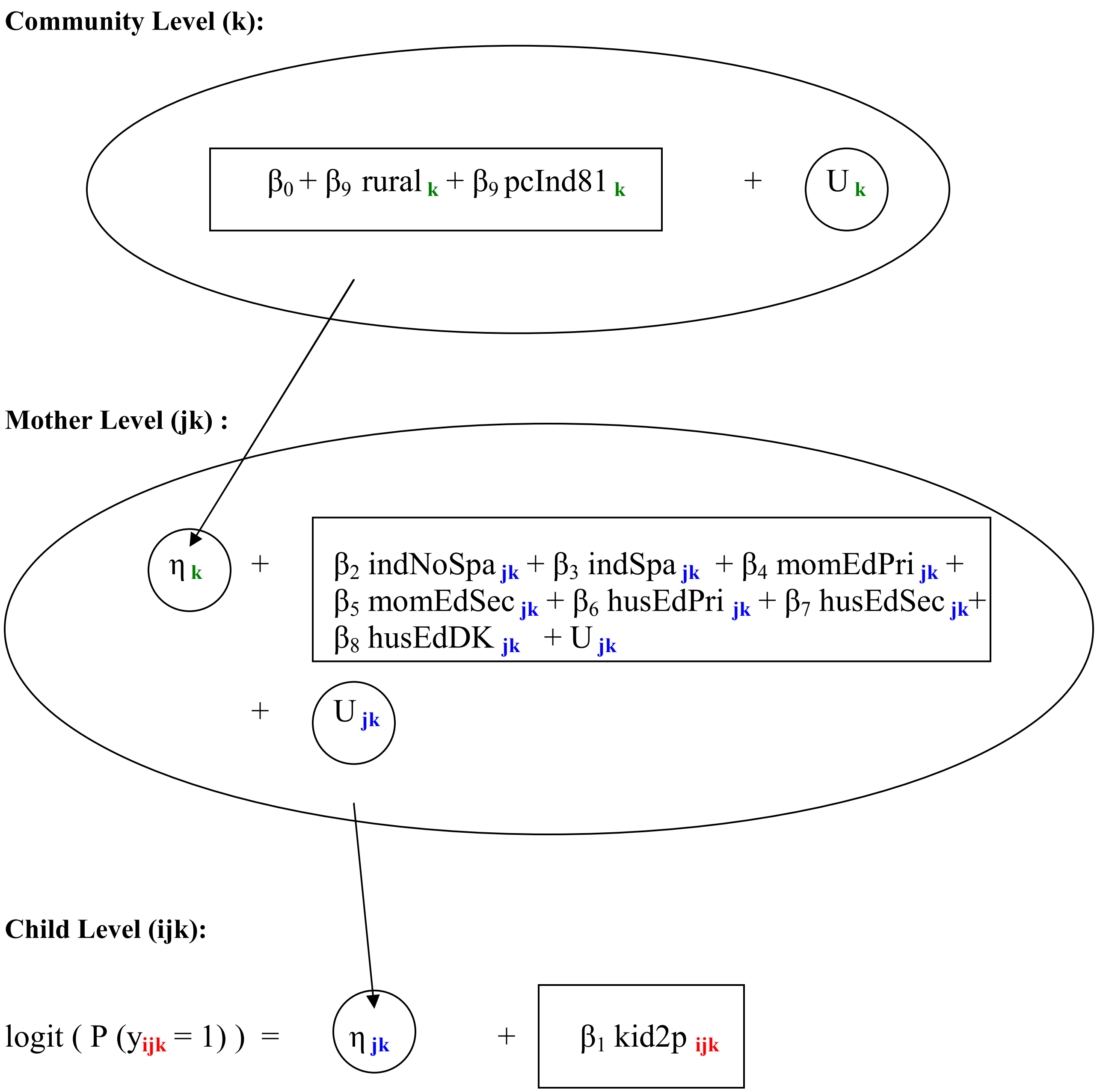
The above model implies:

$$\eta_k\sim \text{Normal}\,(\beta_0+\beta_9\ \text{rural}_k+\beta_9\ \text{pcInd81}_k\ \ ,\,\tau^2{}_2)$$

$$\begin{aligned}\eta_{jk\,|}\,\eta_k\sim \text{Normal}\,(\eta_k+\ & \beta_2\ \text{indNoSpa}_{jk}+\beta_3\ \text{indSpa}_{jk}+\beta_4\ \text{momEdPri}_{jk}+\\ & \beta_5\ \text{momEdSec}_{jk}+\beta_6\ \text{husEdPri}_{jk}+\beta_7\ \text{husEdSec}_{jk}+\\ & \beta_8\ \text{husEdDK}_{jk}\\ ,\,\tau^2{}_1)\end{aligned}$$

So what's the distribution of η_{jk}?

A graphical representation:



Our 3-level random-coefficient model:

$$\log\left(\frac{p(y_{ijk}=1)}{1-p(y_{ijk}=1)}\right)=\eta_{ijk}$$
$$\eta_{ijk}=\beta_0+(\beta_1+U_{k1})kid2p_{ijk}+\beta_2 rural_k+\beta_3 pcInd81_k+U_{jk}+U_{k0}$$

The above model can be decomposed as:

First level (Child):

$$\text{logit}\,(\,P\,(y_{ijk}=1)\,)\;=\eta_{jk}+(\beta_1+U_{k1})\,\text{kid2p}_{ijk}$$

Second level (Mother):

$$\eta_{jk}=\eta_k+U_{jk}$$

$$U_{jk}\sim\text{Normal}\,(\,0,\tau^2_1)$$

Third level (Cluster):

$$\eta_k=\beta_0+\beta_2\ \text{rural}_k+\beta_3\,\text{pcInd81}_k+U_{k0}$$

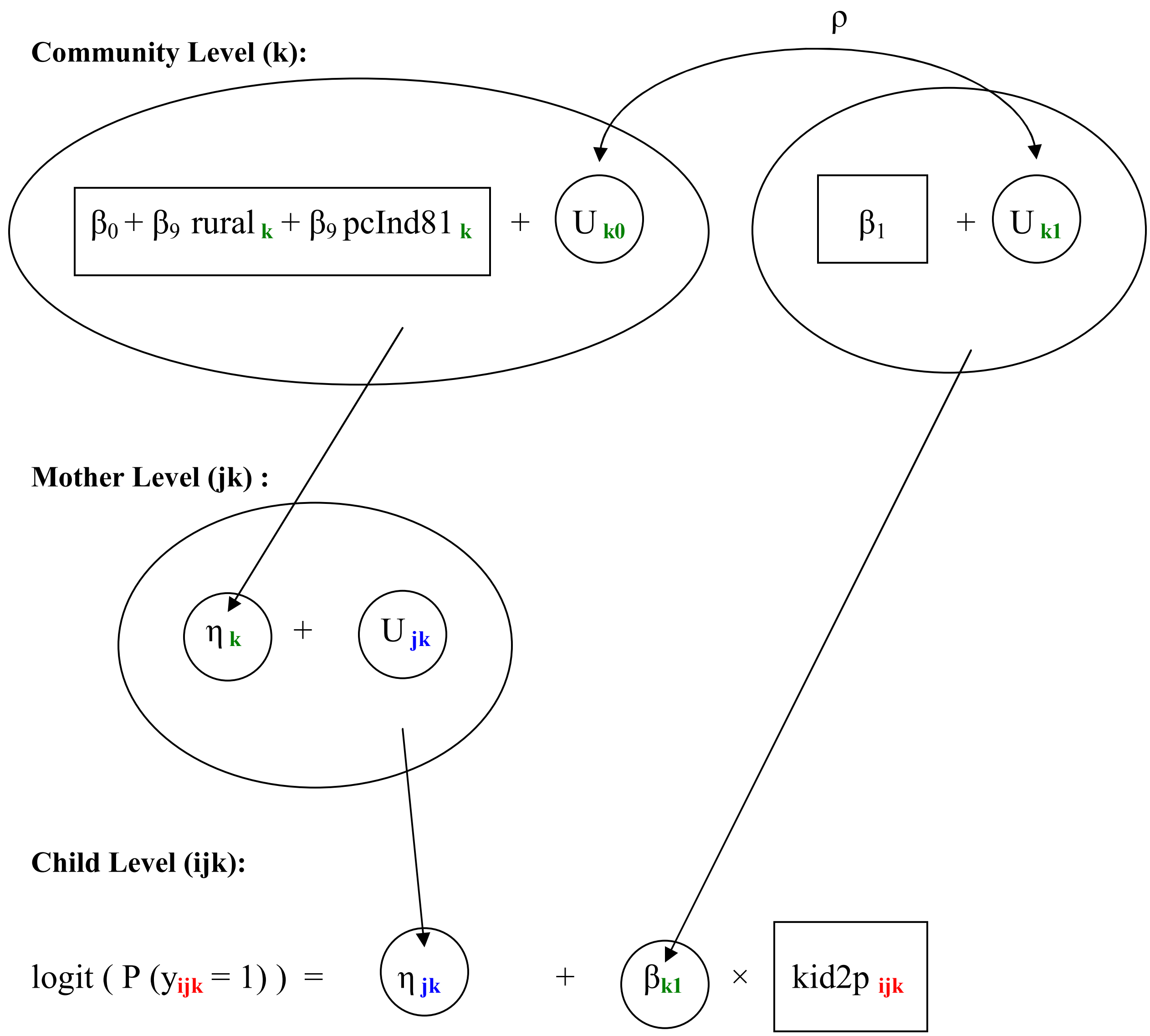
$$U_{k0}\sim\text{Normal}\,(\,0,\tau^2_2)$$
$$U_{k1}\sim\text{Normal}\,(\,0,\tau^2_3)$$
$$\text{cor}\,(\,U_{k0}\,,U_{k1}\,)=\rho$$

The above model implies:

$$\eta_k\sim\text{Normal}\,(\beta_0+\beta_2\ \text{rural}_k+\beta_3\,\text{pcInd81}_k\;,\tau^2_2)$$

$$\eta_{jk}\,|\,\eta_k\sim\text{Normal}\,(\eta_k\,,\tau^2_1)$$

Or Graphically:



X-interaction Model 1:

$$\log\left(\frac{p(y_{ik}=1)}{1-p(y_{ik}=1)}\right)=\eta_{ik}$$

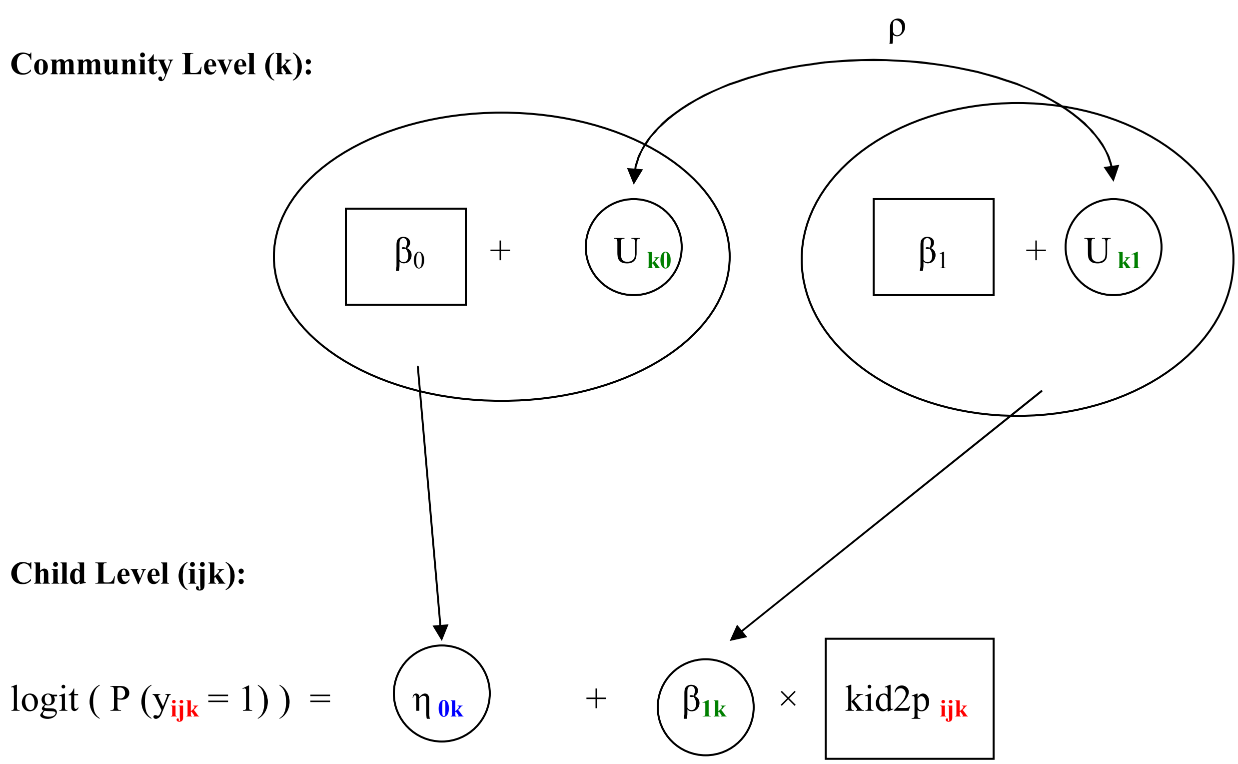
$$\eta_{ik}=\beta_{0k}+\beta_{1k}kid2p_{ik}$$

$$\beta_{0k}=\beta_0+U_{k0}$$

$$\beta_{1k}=\beta_1+U_{k1}$$

Or equivalently:

$$\eta_{ik}=\beta_0+\beta_1 kid2p_{ik}+U_{k0}+U_{k1}kid2p_{ik}$$



X-interaction Model 2:

$$\log\left(\frac{p(y_{ik}=1)}{1-p(y_{ik}=1)}\right)=\eta_{ik}$$

$$\eta_{ik}=\beta_{0k}+\beta_{1k}kid2p_{ik}$$

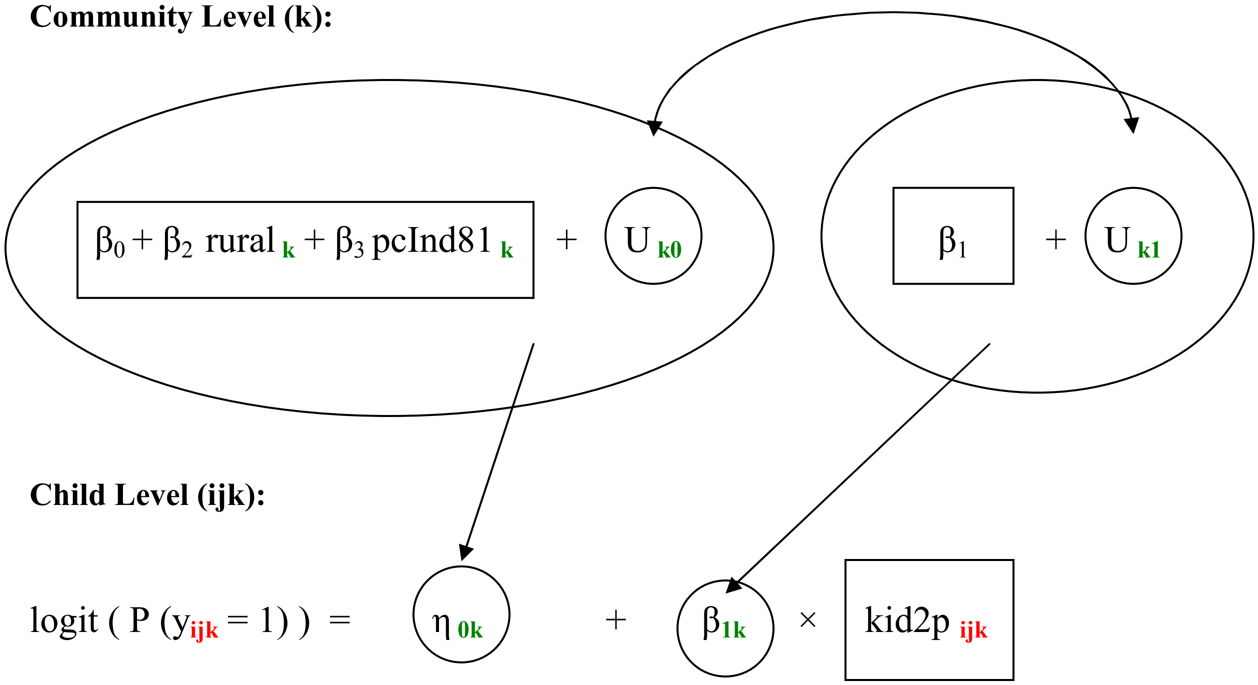
$$\beta_{0k}=\beta_0+\beta_2 rural_k+\beta_3 pcInd81_k+U_{k0}$$

$$\beta_{1k}=\beta_1+U_{k1}$$

Or equivalently:

$$\eta_{ik}=\beta_0+(\beta_1+U_{k1})kid2p_{ik}+\beta_2 rural_k+\beta_3 pcInd81_k+U_{k0}$$

Community Level (k):



X-interaction Model 3:

$$\log\left(\frac{p(y_{ik}=1)}{1-p(y_{ik}=1)}\right)=\eta_{ik}$$
$$\eta_{ik}=\beta_{0k}+\beta_{1k}kid2p_{ik}$$
$$\beta_{0k}=\beta_0+\beta_2 rural_k+\beta_3 pcInd81_k+U_{k0}$$
$$\beta_{1k}=\beta_1+\beta_4 rural_k+\beta_5 pcInd81_k+U_{k1}$$

Community Level (k):