Review: cross-level interactions in logistic regression
1) Model with random intercept and random slope and random effects distributions do not depend on the covariates (latent variable formulation)

\[ y_{ik}^* = \eta_{1k} + \eta_{2k} x_{ik} + \epsilon_{ik} \quad \text{Stage 1} \]

\[ \eta_{1k} = \gamma_1 + \varsigma_{1k} \quad \text{Stage 2} \]

\[ \eta_{2k} = \gamma_2 + \varsigma_{2k} \]

\[ \varsigma_{1k} \sim N(0, \tau_1^2) \]

\[ \varsigma_{2k} \sim N(0, \tau_2^2) \]

\[ y_{ik}^* = \gamma_1 + \varsigma_{1k} + (\gamma_2 + \varsigma_{2k}) x_{ik} + \epsilon_{ik} \]

\[ \varsigma_{1k} \sim N(0, \tau_1^2) \]

\[ \varsigma_{2k} \sim N(0, \tau_2^2) \]
In lab today

**Model 1:** What is the effect of $kid2p_{ik}$ accounting for the between-community heterogeneity?

$$\log \left( \frac{p(y_{ik} = 1)}{1 - p(y_{ik} = 1)} \right) = \eta_{ik}$$

$$\eta_{ik} = \beta_{0k} + \beta_{1k} kid2p_{ik}$$

$$\beta_{0k} = \beta_0 + U_{k0}$$

$$\beta_{1k} = \beta_1 + U_{k1}$$

$\beta_{0k}$: community-specific intercept, i.e., baseline log odds of being immunized (<2y)

$\beta_{1k}$: community-specific slope of $kid2p_{ik}$, i.e., log OR being immunized comparing $\geq 2$y versus <2y.

The equivalent 1-line writing of $\eta_{ik}$ is:

$$\eta_{ik} = \beta_0 + \beta_1 kid2p_{ik} + U_{k0} + U_{k1} kid2p_{ik}$$

$\beta_0$: overall intercept (fixed effects)

$\beta_1$: main effect of $kid2p_{ik}$ (fixed effects)
gllamm model

|-immun-| exp(b)   | Std. Err. | z   | P>|z|  | [95% Conf. Interval] |
|-------|---------|-----------|-----|------|----------------------|
|       | kid2p   | 3.073634  | .4969815 | 6.94 | 0.000  | 2.238823   | 4.219728 |

Variances and covariances of random effects

***level 2 (cluster)

- var(1): 1.2882633 (0.47966448)
- cov(2,1): -0.65561142 (0.39690843) cor(2,1): -0.71194885
- var(2): 0.65824989 (0.36732232)
2) Model with random intercept and random slope and random intercept distribution depends on a level-2 covariate

\[ y_{ik}^* = \eta_{1k} + \eta_{2k} x_{ik} + \epsilon_{ik} \]

\[ \eta_{1k} = \gamma_{11} + \gamma_{12} z_k + \varsigma_{1k} \]

\[ \eta_{2k} = \gamma_2 + \varsigma_{2k} \]

\[ \varsigma_{1k} \sim N(0, \tau_1^2) \]

\[ \varsigma_{2k} \sim N(0, \tau_2^2) \]
Model 2: Does community-level covariates explain the between-community heterogeneity in the baseline log odds of being immunized?

\[
\log \left( \frac{p(y_{ik} = 1)}{1 - p(y_{ik} = 1)} \right) = \eta_{ik} \\
\eta_{ik} = \beta_{0k} + \beta_{1k} kid2 p_{ik} \\
\beta_{0k} = \beta_0 + \beta_2 rural_k + \beta_3 pcInd81_k + U_{k0} \\
\beta_{1k} = \beta_1 + U_{k1}
\]

The equivalent 2-stage writing of \( \eta_{ik} \) is:

\[
\eta_{ik} = \beta_0 + (\beta_1 + U_{k1}) kid2 p_{ik} + \beta_2 rural_k + \beta_3 pcInd81_k + U_{k0}
\]

\( \beta_{0k}, \beta_{1k}, \beta_0, \beta_1 \): Same as above.
\( \beta_2 \): main effect of \( rural_k \) (fixed effects)
\( \beta_3 \): main effect of \( pcInd81_k \) (fixed effects)
gllamm model

|        | exp(b)   | Std. Err. | z     | P>|z|   | [95% Conf. Interval] |
|--------|----------|-----------|-------|-------|----------------------|
| kid2p  | 2.984958 | .4724544  | 6.91  | 0.000 | 2.188826             | 4.070662             |
| rural  | .5294077 | .0867878  | -3.88 | 0.000 | .3839278             | .7300136             |
| pcInd81| .3842638 | .0782185  | -4.70 | 0.000 | .257848              | .5726576             |

Variances and covariances of random effects

***level 2 (cluster)

\[ \text{var(1)}: .85945899 (.36518027) \]
\[ \text{cov(2,1)}: -.4942948 (.33061796) \quad \text{cor(2,1)}: -.68798101 \]

\[ \text{var(2)}: .60061203 (.34310316) \]

The variance of the random intercept decrease, indicating that the community-level covariates \( r_{ural} \) and \( pc\text{Ind}81 \) explain the between-community variability in baseline log odd of being immunized. The statistical significance of the main effects of \( r_{ural} \) and \( pc\text{Ind}81 \) also suggests this conclusion.
3) Model with random intercept and random slope and random effects distributions depend on a level-2 covariate

\[
\begin{align*}
\eta_{1k}^* &= \eta_{1k} + \eta_{2k} x_{ik} + \epsilon_{ik} \\
\eta_{1k} &= \gamma_{11} + \gamma_{12} z_k + \varsigma_{1k} \\
\eta_{2k} &= \gamma_{21} + \gamma_{22} z_k + \varsigma_{2k} \\
\varsigma_{1k} &\sim N(0, \tau_1^2) \\
\varsigma_{2k} &\sim N(0, \tau_2^2)
\end{align*}
\]

**Stage 1**

\[
\begin{align*}
\eta_{1k}^* &= \gamma_{11} + \varsigma_{1k} + (\gamma_{21} + \varsigma_{2k}) x_{ik} + \gamma_{12} z_k + \gamma_{22} z_k x_{ik} + \epsilon_{ik} \\
\varsigma_{1k} &\sim N(0, \tau_1^2) \\
\varsigma_{2k} &\sim N(0, \tau_2^2)
\end{align*}
\]
Model 3: Does community-level covariates explain the between-community heterogeneity in both the baseline log odds of being immunized and the log OR being immunized comparing $\geq 2y$ versus $<2y$?

$$\log\left(\frac{p(y_{ik} = 1)}{1 - p(y_{ik} = 1)}\right) = \eta_{ik}$$

$$\eta_{ik} = \beta_{0k} + \beta_{1k} \cdot kid2p_{ik}$$

$$\beta_{0k} = \beta_0 + \beta_{2rural_k} + \beta_{3pcInd81_k} + U_{k0}$$

$$\beta_{1k} = \beta_1 + \beta_{4rural_k} + \beta_{5pcInd81_k} + U_{k1}$$

The equivalent 2-stage writing of $\eta_{ik}$ is:

$$\eta_{ik} = \beta_0 + \beta_{2rural_k} + \beta_{3pcInd81_k} + U_{k0} + (\beta_1 + \beta_{4rural_k} + \beta_{5pcInd81_k} + U_{k1}) \cdot kid2p_{ik}$$

$$\eta_{ik} = \beta_0 + \beta_{1kid2p_{ik}} + \beta_{2rural_k} + \beta_{3pcInd81_k} + \beta_{4rural_k} \cdot kid2p_{ik} + \beta_{5pcInd81_k} \cdot kid2p_{ik} + U_{k0} + U_{k1} \cdot kid2p_{ik}$$

$\beta_{0k}, \beta_{1k}, \beta_0, \beta_1, \beta_2, \beta_3$: Same as above.

$\beta_4$: cross-level interaction between $rural_k$ and $kid2p_{ik}$ (fixed effects)

$\beta_5$: cross-level interaction between $pcInd81_k$ and $kid2p_{ik}$ (fixed effects)
. gllamm immun kid2p rural pcInd81 int_2p_ru int_2p_pc, family(binomial) link(logit) i(cluster) nrf(2) eqs(inter slope) nip(4 4) adapt eform

gllamm model

|        | exp(b)   | Std. Err. |      z |   P>|z|    | [95% Conf. Interval] |
|--------|----------|-----------|-------|--------|---------------------|
| kid2p  | 2.311586 | .7539445  | 2.57  | 0.010  | 1.219784            | 4.380635 |
| rural  | .5115291 | .1639525  | -2.09 | 0.036  | .2729278            | .9587223 |
| pcInd81| .2402431 | .0980197  | -3.50 | 0.000  | .1079839            | .534494  |
| int_2p_ru| 1.045638 | .3464314  | 0.13  | 0.893  | .5462218            | 2.001676 |
| int_2p_pc| 1.755981 | .727406   | 1.36  | 0.174  | .7796755            | 3.95481  |

Variance and covariances of random effects

***level 2 (cluster)

var(1): .95682725 (.39271689)
cov(2,1): -.56582712 (.34798787) cor(2,1): -.72621719

var(2): .63445517 (.34985769)

The variance of the random slope remains approximately the same, indicating that the community-level covariates rural
d_k and pcInd81_k do not explain the between-community variability in the log OR being immunized comparing >=2y versus <2y. This can be also inferred from the non-statistically significant (cross-level) interaction between kid2p_i_k and the community-level variables rural_k and pcInd81_k.