Classification Using Boolean Functions

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Introduction and Motivation

$X_1, \ldots, X_k$ are 0/1 (False/True) predictors.

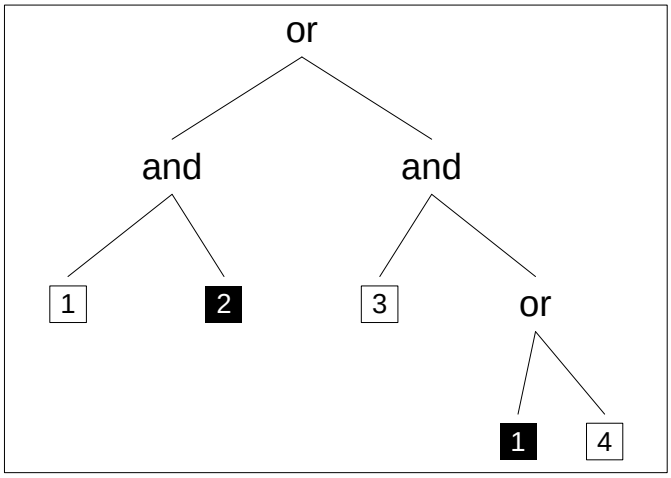
Y is a response variable.

Fit a model $g(F(Y)) = \beta_0 + \sum_{j=1}^{t} \beta_j \times L_j,$ where L_j is a Boolean combination of the covariates, e.g. $L_j = (X_1 \vee X_2) \wedge X_4^c$.

Determine the logic terms L_j and estimate the β_j simultaneously.

Logic Trees

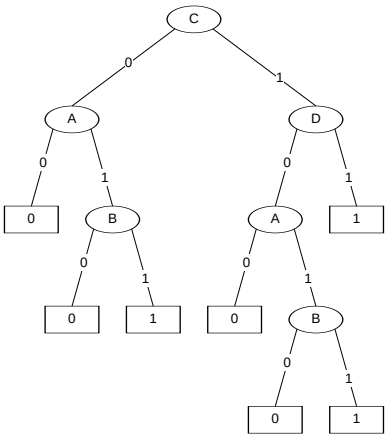
An equivalent representation of $(X_1 \wedge X_2^c) \vee (X_3 \wedge (X_1^c \vee X_4))$ is th following:



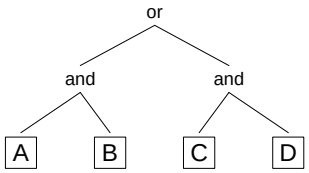
This is a Logic Tree!

Comparison to Decision Trees

Decision Tree



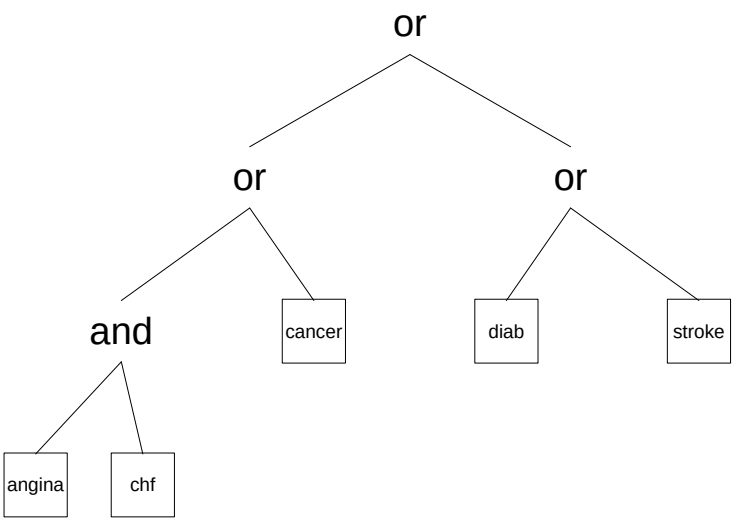
Logic Tree



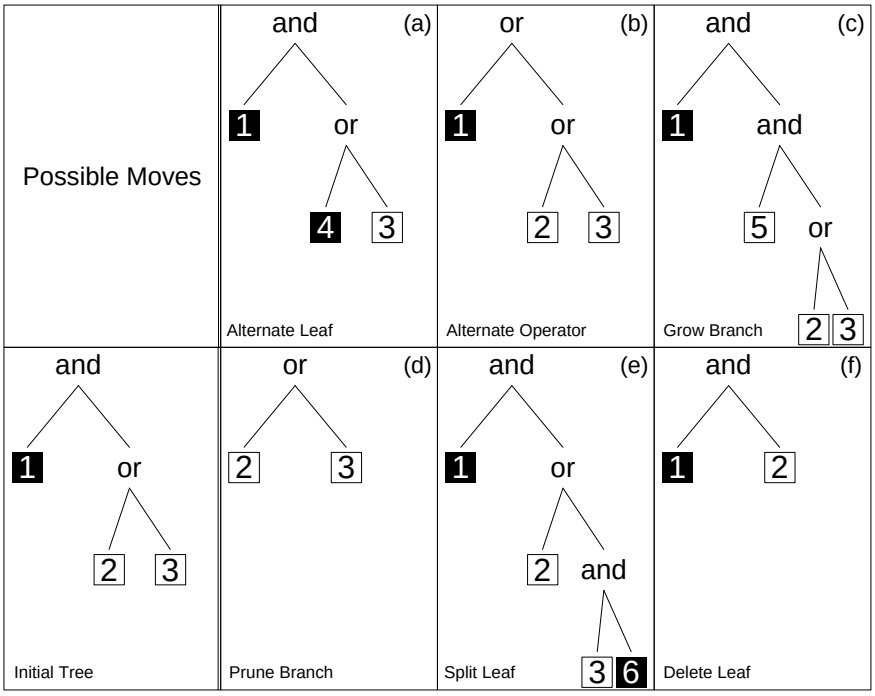
A Decision Tree (CART) is something different!

An Example

$$\text{logit}(\text{death}) = -9.01 + 0.06 \times \text{age} + 1.07 \times \text{L}$$



The Move Set



Simulated Annealing for Logic Regression

We try to fit the model $\quad g(F(Y)) = \beta_0 + \sum_{j=1}^{t} \beta_j \times L_j.$

- Select a scoring function (RSS,log-likelihood, . . .).
- Pick the maximum number of Logic Trees.
- Pick the maximum number of leafs in a tree.
- Initialize the model with $L_j = 0$ for all j.
- Carry out Simulated Annealing Algorithm:
  - Propose a move.
  - Accept or reject the move, depending on scores and temperature scheme.

Simulated Annealing for Logic Regression (2)

- We use the acceptance function $\alpha(\epsilon_{old}, \epsilon_{new}, t) = \min\{1, \exp([\epsilon_{old} - \epsilon_{new}]/t)\}$
- We run homogeneous Markov chains at constant temperatures.
- We constructed the move set to be irreducible and aperiodic, therefore each homoge neous Markov chain has a limiting distribution $\pi_t(S)$.
- The limit (as $t \to 0$) of those distributions assigns probability 1 to the optimal scorin states.
- With limited resources, we cannot guarantee to find an optimal scoring state.

Another Example

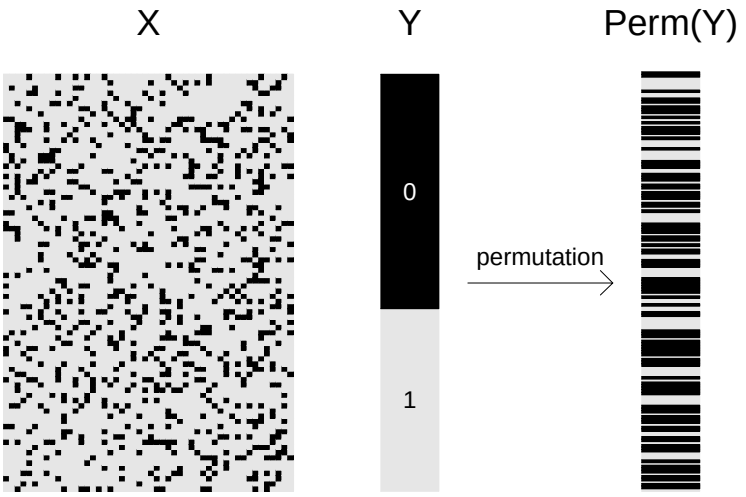
The Cardiovascular Health Study

(Fried et. al. , Annals of Epidemiology, 1991).

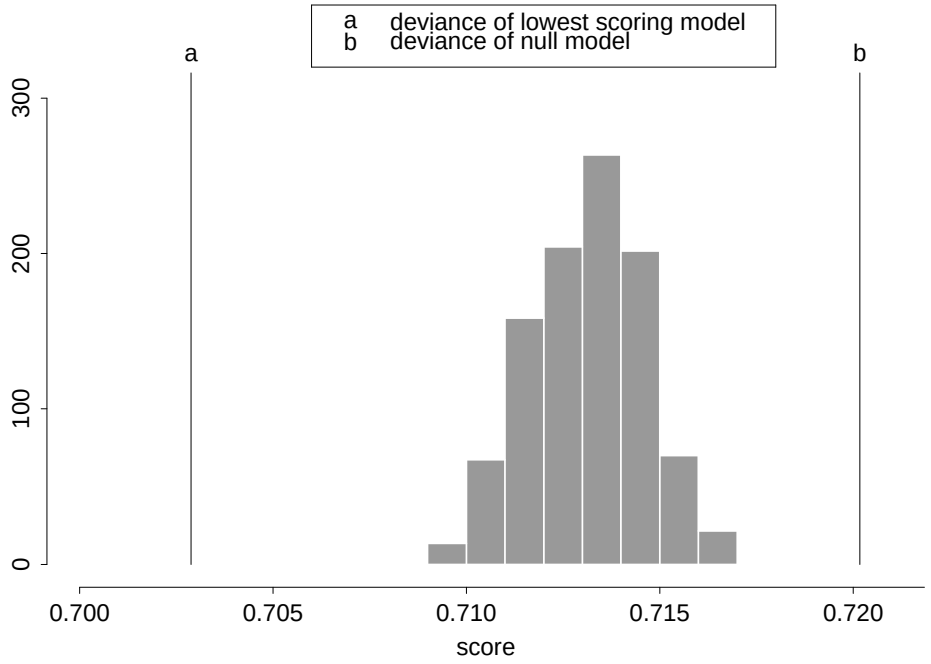
- The Cardiovascular Health Study (CHS) is a study of coronary heart disease and strok in elderly people.
- Between 1989 and 1993, 5888 subjects over the age of 65 were recruited in four com munities in the United States.
- During 1992 and 1994, a subset of these patients underwent an MRI scan.
- For 3647 CHS participants, MRI detected strokes (infarcts bigger than 3mm that led t deficits in functioning) were recorded as entries into a 23 region atlas of the brain.
- The mini-mental state examination is a brief screening test for dementia. The respons Y is a variable derived by transforming the mini-mental score.

We investigated models of the form $Y = \beta_0 + \beta_1 \times L_1 + \cdots + \beta_p \times L_p + \epsilon$.

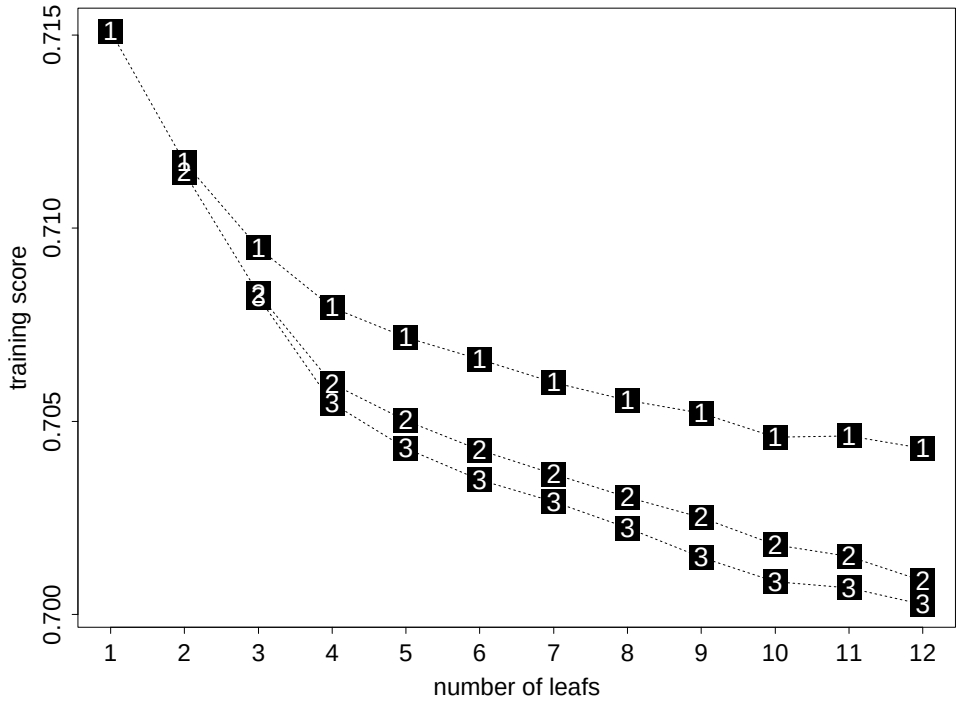
A Global Randomization Test of Association



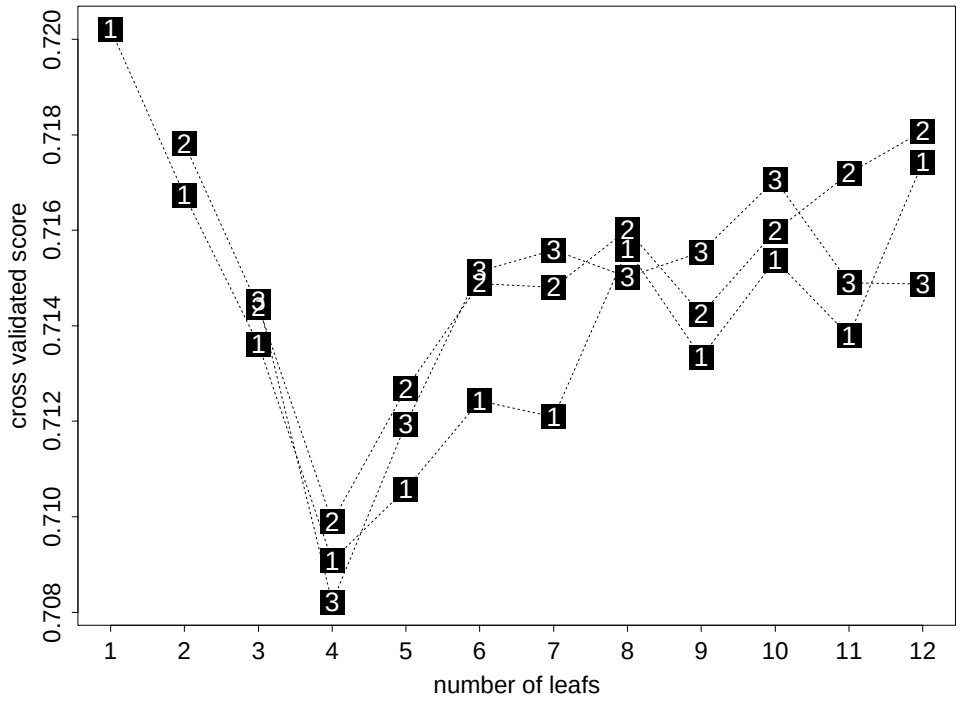
Example Cont.



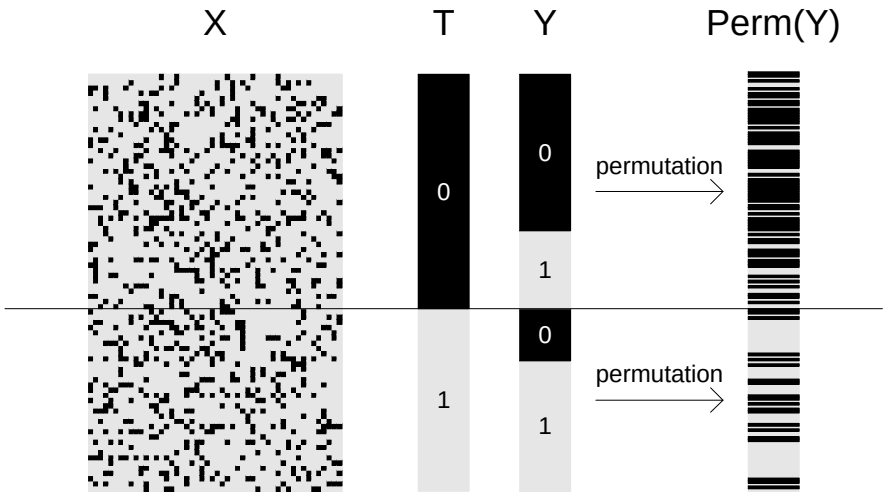
Cross-Validation



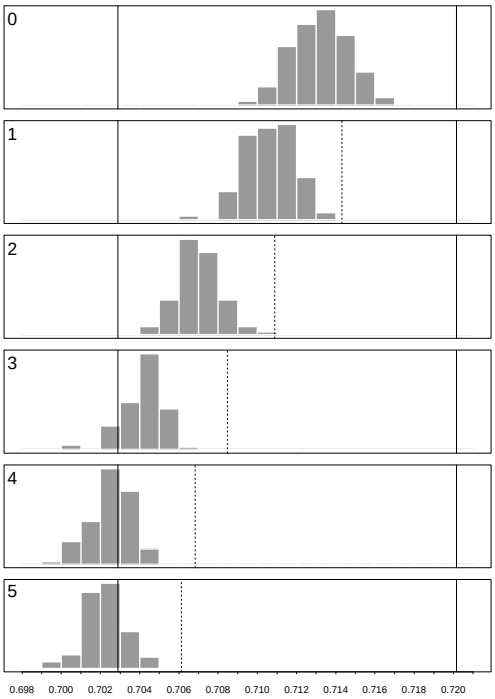
Cross-Validation Cont.



A Sequential Randomization Test for Model Size

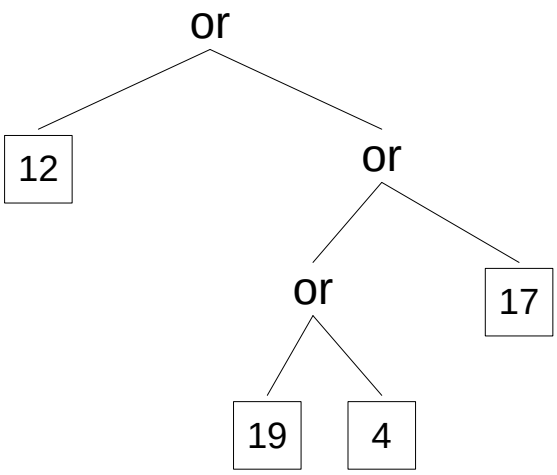


Randomization Test Cont.



Results

The model we found was $Y = 1.96 + 0.36 \times L$, with the following Logi Tree:



Logic vs Linear Model vs MARS

Linear model:

	$\hat{\beta}$	$\hat{\sigma}$	t-value	p-value
Intercept	1.961	0.015	133.98	<0.001
Region 4	0.524	0.129	4.06	<0.001
Region 12	0.460	0.112	4.09	<0.001
Region 17	0.236	0.057	4.17	<0.001
Region 19	0.611	0.157	3.89	<0.001

Logic model:

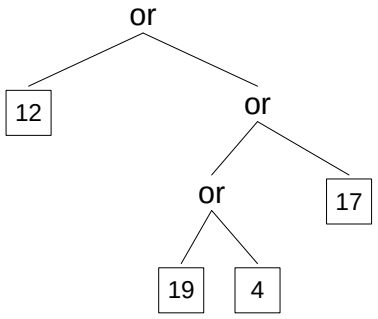
$$Y = 1.96 + 0.36 \times I_{\{X_4 \vee X_{12} \vee X_{17} \vee X_{19} \text{ is true}\}}$$

MARS:

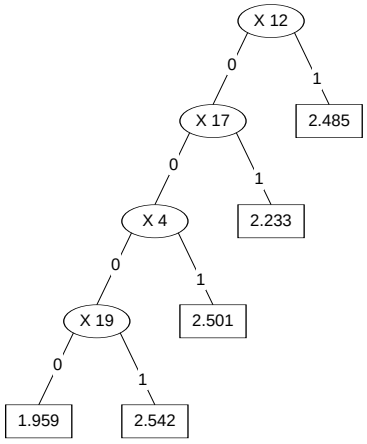
$$Y = 1.96 + 0.53\, X_4 + 0.37\, X_{12} + 0.24\, X_{17} + 0.61\, X_{19} + 1.05\, (X_{12} * X_{15})$$

Logic vs CART

Logic Tree



CART Tree



References

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