Handout 3

Assume that you have three independent measurements $X_1, X_2, X_3$, with $\text{var}(X_i) = \sigma^2$.

It follows that $\text{var}(\bar{X}) = \sigma^2/3$.

Assume that you have to subtract a baseline $B$ from each measurement. What is $\text{var}(X_i - B)$?

If $B$ is just a constant, then $\text{var}(X_i - B) = \text{var}(X_i) = \sigma^2$.

If $B$ is a measurement with $\text{var}(B) = \sigma_B^2$, then $\text{var}(X_i - B) = \text{var}(X_i) + \text{var}(B) = \sigma^2 + \sigma_B^2$.

What is the variance of the average of those values, i.e. what is the variance of $\sum_i (X_i - B) / 3$ .. ?

$$\text{var}\left(\frac{\sum_i (X_i - B)}{3}\right) = \text{var}\left(\frac{1}{3}(X_1 + X_2 + X_3 - 3 \times B)\right) = \text{var}(\bar{X} - B) = \text{var}(\bar{X}) + \text{var}(B) = \frac{\sigma^2}{3} + \sigma_B^2.$$ 

If each experiment has its own baseline $B_i$, measured independently, with $\text{var}(B_i) = \sigma_B^2$, then

$$\text{var}\left(\frac{\sum_i (X_i - B_i)}{3}\right) = \text{var}(\bar{X} - \bar{B}) = \text{var}(\bar{X}) + \text{var}(\bar{B}) = \frac{\sigma^2}{3} + \frac{\sigma_B^2}{3} = \frac{\sigma^2 + \sigma_B^2}{3}.$$