15 Introduction to Design and Analysis of Experiments

Experiments are usually run for one or more of the following reasons:

- To compare responses achieved at different settings of controllable variables.
- To determine the principal causes of variation in a measured response.
- To find the conditions that give rise to a maximum and minimum response.
- To obtain a mathematical model in order to predict future responses.

Fundamental to experimental design are replication, blocking, and randomization, discussed throughout the notes. Replication and blocking increase the precision in the experiment, randomization decreases the bias.

15.1 Definition: A source of variation is anything that could cause an observation to have a different numerical value from another observation. Sources of variation can be divided into two classes: a treatment factor is a source of variation whose effect on the data is to be studied; a nuisance factor is a source of variation that is of no particular interest to the experimenter. An experimental unit is the item to which the levels of the treatment factors are applied. Here, levels mean the types or amounts of the treatment factor that will be used in the experiment. The experimental design is the rule which specifies which experimental units are to be observed under which treatment levels.

15.2 Definition: A completely randomized design is a design in which the experimenter assigns the experimental units to the treatments completely at random, subject only to the number of observations to be taken on each treatment.

15.3 Example: Coagulation times of blood obtained from mice randomized to different diets were observed. We are mainly interested in finding out whether or not coagulation times differ between treatment groups.

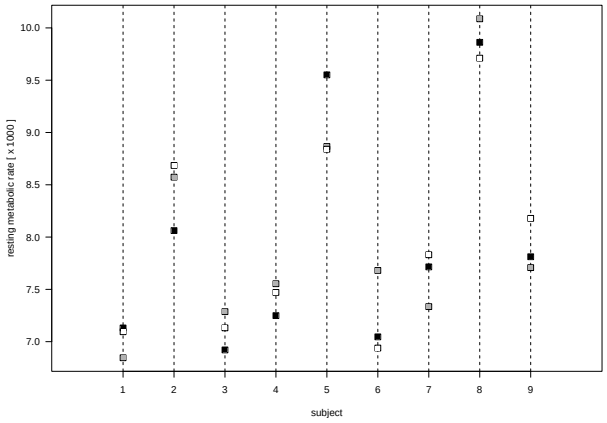
Diet	Coagulation Time							
A	62	60	63	59				
B	63	67	71	64	65	66		
C	68	66	71	67	68	68		
D	56	62	60	61	63	64	63	59

The model could be of the form: Response = Grand Mean + Treatment Effect + Error.

15.4 Definition: A block design is a design in which the experimenter partitions the experimental units into blocks, determines the allocation of treatment to blocks, and assigns the experimental units within each block to the treatments completely at random. If the number of treatments observed are the same in each

block, the design is called a complete block design. A complete block design whose blocks contain a single observation on each treatment are called randomized (complete) block design. When the block size is smaller than the number of treatments and it is therefore not possible to observe every treatment in each block, the design is called an incomplete block design.

15.5 Example: An experiment was run to compare the effects of in-patient and out-patient protocols on the lab measurements of resting metabolic rate in humans. The plot below shows the resting metabolic rates of 9 subjects for three different protocols (indicated by the white, gray and black color), applied to each person. Here, the subjects serve as blocks.



The model could be of the form: Response = Grand Mean + Protocol Effect + Block effect + Error.

15.6 Definition: When an experiment involves two major sources of variation, the factors are either crossed or nested. If the factors are crossed, experimental units are used from all possible combinations of the levels of the factors. When the factors are nested, a particular level of one of the factors occurs at only one level of the other factor.

15.7 Example: Two independent measurements were taken on a total of twelve mosquitoes, four mosquitoes within each cage. Here, the mosquitoes are nested within the cages.

Cage 1				Cage 2				Cage 3			
1	2	3	4	5	6	7	8	9	10	11	12
58.5	77.8	84.0	70.1	69.8	56.0	50.7	63.8	56.6	77.8	69.9	62.1
59.5	80.9	83.6	68.3	69.8	54.5	49.3	65.8	57.5	79.2	69.2	64.5

The model could be: Response = Grand Mean + Cage Effect + Mosquito w/in cage effect + Error.

15.8 Example: The table below shows the fat consumption of rats. Six male and six female rats were randomized to be fed either fresh or rancid lard, with three rats of each gender in each treatment group.

	Fresh lard			Rancid lard		
Male	709	679	699	592	538	476
Female	657	594	677	508	505	539

The model could be of the form: Response = Grand Mean + Treatment Effect + Gender effect + Error.

15.9 Note: None of the models suggested in the above examples have considered the important concept of interaction. We will discuss this later in detail.

15.10 Note: There are studies in which it is not possible in practice to assign the experimental units to the treatments. For example, if we are interested in the effect of smoking on lung cancer, it is unethical to assign the treatment level "smoking" to a subject. The association of smoking and lung cancer can only be investigated in the framework of an observational study, recording smoking habits and cancer incidences in a population. This is not an experiment, and although many of the techniques discussed in the following can be used for analysis, the cause and effect conclusions we can draw from an experiment are not valid.

15.11 Note: There are studies in which we are interested how subpopulations defined by some given characteristics differ. For example, the table below shows the mean weights of 3 genotypes of beetles, reared at a density of 20 beetles per gram of flour. Four series of experiments represent blocks. We are interested in drawing inference whether or not weights differ for the three genotypes in the population.

	genotype		
block	++	+b	bb
1	0.958	0.986	0.925
2	0.971	1.051	0.952
3	0.927	0.891	0.829
4	0.971	1.010	0.955

Strictly speaking, this is not an experiment since the experimental units (the beetles) were not randomized to the factor of interest (the genotype). If the beetles were randomly sampled from the respective populations, we can still draw some inference about the populations using the same analysis as for a complete block design.