The Generalized 2D-PSWF Method for Tracking Dynamic Signal with High Temporal Resolution

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INTRODUCTION

The two-dimensional prolate spheroidal wave function (2D-PSWF) method [1-3] offers an efficient way of trading-off between spatial and temporal resolution for dynamic MRI, with minimal penalty due to truncation and partial volume effects. To efficiently reduce the number of required data points, the 2D-PSWF method tailors the k-space sampling area according to the size and shape of the predetermined ROI and creates a matching 2D-PSWF filter to optimally reduce truncation effects. In this method, the spatial information in the reduced k-space data is used to calculate the total image intensity over a non-square ROI instead of producing a low-resolution image. This method can be used for tracking dynamic signals from non-square ROIs using a reduced k-space sampling area, while achieving minimal signal leakage. This report presents a generalized method that allows for application of the 2D-PSWF method to an arbitrary k-space trajectory. An implementation of this method to a spiral trajectory is demonstrated with a high temporal resolution fMRI study.

THEORY

The 2D-PSWF method calculates the optimal sampling region, ***A***, of a predetermined size *a*, which maximizes the total signal over an ROI, ***B***, in image-space and combines this with a matched two-dimensional filter that maximizes the energy concentration in ***B***. The key to the method is finding the matched filter function g(**k**), that vanishes off ***A*** and whose inverse Fourier transform, G(**x**), has maximal signal concentration in ***B***. Once g(**k**) is obtained, the total signal intensity over the ROI ***B*** can be evaluated directly from the reduced k-space area. The original 2D-PSWF theory was developed only for the rectilinear sampling case [1-3]. To increase its suitability for dynamic MRI and CSI studies, a generalized 2D-PSWF theory is presented that can be applied to non-rectilinear data acquisition methods. This generalization is important, as k-space is typically sampled in a non-rectilinear fashion for dynamic MRI studies, due to hardware limitations.

The main difference between the original and generalized method lies in the calculation of the filter function g(**k**) (Step 3 in the outline to the left). In the notation of the generalized 2D-PSWF two operators are defined:

$$\mathbf{TF} = \sum_{\mathbf{x}\in\Omega} e^{-i\frac{2\pi}{N}(\mathbf{x},\mathbf{k})} F(\mathbf{x})$$

and

$$\mathbf{T}^{-1}\mathbf{F} = \sum_{\mathbf{k}_j\in A} e^{i\frac{2\pi}{N}(\mathbf{x},\mathbf{k}_j)} f(\mathbf{k}_j)$$

It can be shown that for non-uniform data points the optimal PSWF filter is given by the eigenvector that corresponds to the largest eigenvalue λ of the matrix

$$K_{B,A} = \mathbf{I}_B\mathbf{T}^{-1}\mathbf{R}^{-2}\mathbf{T}\mathbf{I}_B \qquad [1]$$

where $\mathbf{R}=(\mathbf{TT}^{-1})^{1/2}$ and the matrix $\mathbf{TI}_B$ is defined as the *b* columns of the matrix **T** that correspond with elements in the ROI *B*. Once the largest eigenvector, **η**, of this matrix is obtained, we can use it to calculate the optimal filter,

$$\mathbf{g} = \frac{1}{\lambda}\mathbf{R}^{-1}\mathbf{TI}_B\boldsymbol{\eta} \qquad [2]$$

with the desired properties.

The 2D-PSWF Method

1. Choose the FOV and the image dimension *N*, and design the shape and size of the ROI ***B***.
2. Determine the optimal sampling region $\boldsymbol{A}=\{k_i| i=1,\ldots a\}$ corresponding to the choice of *N* and *B* (see [1-3] for more details).
3. Calculate the matrix $\mathbf{K}_{B,A}$ and the optimal filter function **g** (Eqs. 1&2).
4. Apply the filter, **g**, to the k-space data f(**k**,t) to determine the total signal over the ROI at time *t*, i.e. calculate:
$$F_B(t) = \sum_i f(k_i,t)g(k_i) \qquad t=1,\ldots T$$
5. Translate the ROI using a phase shift to track the signal in different regions.

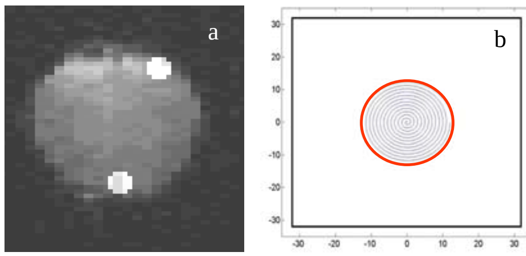

Figure 1. (a) The two ROIs in the motor (upper) and visual (lower) cortices superimposed on a low-resolution image of the acquired slice. For a FOV of 240 *mm* and a 64×64 image resolution, the size of the ROI corresponds to a region consisting of 21 adjacent voxels in image-space. (b) The optimal sampling region, *A*, corresponding to the ROI given in (a) is marked by the red circle. The spiral trajectory with 3628 points used to sample *A* is superimposed. Compare the sampled subset with the extent of k-space needed to fully reconstruct a 64×64 image which is marked by the black box.

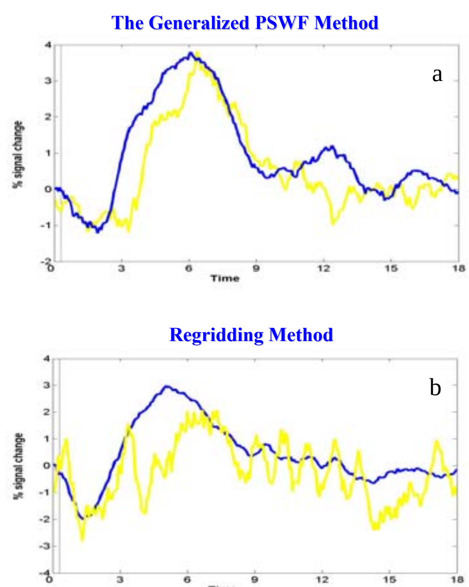

Figure 2. The dynamic signal change over the visual (blue) and motor (yellow) cortices for the first 18 s following visual stimuli using (**a**) the generalized PSWF method and (**b**) signal averaging over the ROIs from images reconstructed using the traditional regridding approach. A delay of approximately 300 *ms* in time-to-peak between the two curves is clearly demonstrated in plot **a.**

REFERENCES

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METHODS

The experimental data was collected on a 3.0 T whole body scanner (GE magnet, General Electric Medical Systems, Milwaukee, WI, USA). The images were acquired in an oblique slice containing both primary visual and motor cortices using spiral trajectories with TR 60 ms, TE 30 ms, flip angle 15 degrees, field of view 240×240 mm^2, slice thickness 10.3 mm, matrix 24×24 and bandwidth 125 kHz.

The fMRI experiment was designed to use the generalized 2D-PSWF method to simultaneously track the hemodynamic signals in the visual and motor cortices while the subject undergoes a visual-motor activation paradigm. The activation paradigm consisted of six cycles of 30 s intervals. At the beginning of each interval a 100 ms light flash was presented. The subject was instructed to press a button immediately after the appearance of the flash, thereby leading to activation of the motor cortex. During the 30 second interval, 500 images were acquired one every 60 ms. The sequence was repeated six times, each time producing a new sequence of 500 time points.

Following the outline of the generalized 2D-PSWF method, two circular regions of interest (ROI) with a radius of 8 *mm* were chosen with one ROI placed in the primary visual cortex and the other placed in the primary motor cortex (Fig. 1a). To calculate the matrix $\mathbf{K}_{B,A}$, the ROI *B* needs to be rewritten in terms of the coordinates of the voxels contained within the region. The number of voxels contained in *B* depends on the size and shape of the ROI, as well as the FOV and the image resolution, *N* (chosen by the experimenter). For a FOV of 240 *mm* and *N*=64, *B* consists of 21 adjacent voxels in image-space. To fully reconstruct a 64×64 image we would need to sample 4096 points in k-space, spaced a distance FOV^{-1} from one another. However, since we are solely interested in the signal over *B*, the 2D-PSWF method allows us to sample a subset of k-space that is optimally tailored to the ROI. As the ROI were chosen to be circles, the optimal sampling in k-space is also a circular region [3]. The optimal sampling region corresponding to the choice of FOV, *N* and *B* is shown in red in Fig 1b. This k-space sub region was sampled once every 60 ms, and the resulting data was used to measure brain activity in the ROI. The k-space data was sampled using a spiral trajectory (Fig. 1b), consisting of 3628 points, as this is the most time-efficient way to sample a circular region of k-space.

RESULTS

Applying the generalized 2D-PSWF method to the experimental data, time series plots were obtained showing the dynamic signal change over each of the two ROI. Fig. 2a shows a plot of the signal change in the visual cortex (blue line) and the motor cortex (yellow line) averaged over the six runs. Typically under conditions needed to increase the temporal resolution, tracking the hemodynamic responses is difficult because the image SNR is significantly attenuated by the drastically reduced TR. But, as seen in Fig. 2a, the 2D-PSWF method provides adequate SNR for dynamic studies. With the increased temporal resolution, we were able to precisely determine the delay in motor activation onset (approximately 300 *ms*). For comparison purposes Fig. 2b shows a similar plot obtained using images reconstructed with a conventional re-gridding method, where the signal is obtained by averaging over the ROIs. It is clear that the motor activation is significantly attenuated as a result of partial volume effects.

The experimental results presented in the report demonstrate that the 2D-PSWF method can be a valuable tool for the studies of functional neuron-neuron interaction, synchronization and connectivity.