

Simpson's paradox

Victim	Defendant	Death penalty		
		yes	no	% yes
White	White	53	414	11.3
	Black	11	37	22.9
Black	White	0	16	0.0
	Black	4	139	2.8
		53	430	11.0
		15	176	7.9
White		64	451	12.4
Black		4	155	2.5

Discussion

- Marginally, white defendants received the death penalty a greater percentage of time than black defendants
- Across white and black victims, black defendant's received the death penalty a greater percentage of time than white defendants
- Simpson's paradox refers to the fact that marginal and conditional associations can be opposing
- The death penalty was enacted more often for the murder of a white victim than a black victim. Whites tend to kill whites, hence the larger marginal association.

Confounding

- Variables that are correlated with both the explanatory and response variables can distort the estimated effect

Victim's race was correlated with defendant's race and death penalty

- One strategy to adjust for confounding variables is to **stratify** by the confounder and then combine the strata-specific estimates

Requires appropriately weighting the strata-specific estimates

- Unnecessary stratification reduces precision

Mantel Haenszel estimator

Let n_{ijk} be entry i, j of table k

The k^{th} sample odds ratio is $\hat{\theta}_k = \frac{n_{11k}n_{22k}}{n_{12k}n_{21k}}$

The Mantel Haenszel estimator is of the form

The weights are

The estimator simplifies to

SE of the log is given in Agresti (page 235) or Rosner (page 656)

	Center															
	1		2		3		4		5		6		7		8	
Trt	S	F	S	F	S	F	S	F	S	F	S	F	S	F	S	F
Drug	11	25	16	4	14	5	2	14	6	11	1	10	1	4	4	2
Ctrl	10	27	22	10	7	12	1	16	0	12	0	10	1	8	6	1
n	73		52		38		33		29		21		14		13	

S - Success, F - failure

Drug - Active Drug, Ctrl - placebo

CMH test

- $H_0 : \theta_1 = \dots = \theta_k = 1$ versus $H_a : \theta_1 = \dots = \theta_k \neq 1$
- The CHM test applies to other alternatives, but is most powerful for the H_a given above
- Same as testing conditional independence of the response and exposure given the stratifying variable
- CMH conditioned on the rows and columns for each of the k contingency tables resulting in k hypergeometric distributions and leaving only the n_{11k} cells free

CMH test cont'd

- Under the conditioning and under the null hypothesis
- The CMH test statistic is
- For large sample sizes and under H_0 , this test statistic is $\chi^2(1)$ (regardless of how many tables you are summing up)

In R

```
dat <- array(c(11, 10, 25, 27, 16, 22, 4, 10,  
              14, 7, 5, 12, 2, 1, 14, 16,  
              6, 0, 11, 12, 1, 0, 10, 10,  
              1, 1, 4, 8, 4, 6, 2, 1),  
            c(2, 2, 8))
```

```
mantelhaen.test(dat, correct = FALSE)
```

Results: $CMH_{TS} = 6.38$

P-value: .012

Test presents evidence to suggest that the treatment and response are not conditionally independent given center

Some final notes on CMH

- It's possible to perform an analogous test in a random effects logit model that benefits from a complete model specification
- It's also possible to test heterogeneity of the strata-specific odds ratios
- Exact tests (guarantee the type I error rate) are also possible `exact = TRUE` in R