Summary: When studying a group of individuals over time there exists an inclination to explore the data and report results with composite summary measures that collapse the time component. An example of this would be “Those with sleep apnea spent 18% of their total sleep time in REM, whereas those without sleep apnea spent 21%.” Although these composite summary measures are convenient, they do obscure any time-related effects. History Matrix Visualization (HMV) is a technique that can allow the exploration and guide the analysis of longitudinal categorical data, and may provide insights into transitional processes that may have been otherwise obscured when implementing traditional composite summary measures. The HMV method is demonstrated using data of 59 matched apneic and non-apneic individuals from the Sleep Heart Health Study.

Keywords: History matrix; History matrix visualization; Longitudinal categorical data; Markov process; Multi-state survival analysis.
1. Introduction

Transitional data, whether within the framework of Markov Chains where the transitions are modeled as moving from state $A$ to state $B$ or within the context of Survival Analysis where a subject experiences the passage of time before an event occurs, can be used to model various phenomena (Marshall and Jones, 1995; Kay, 1986; Therneau and Grambsch, 2001). The initial concept for the visualization of this data is quite easy to grasp given rudimentary knowledge of matrices and standard computer programming functions such as sort and image available in many programming languages and packages.

History Matrix Visualization (HMV) is a technique that can allow the exploration and guide the analysis of longitudinal categorical data. The process is depicted in Figure 1. Information is extracted from 3 individual hypnograms into history rows. History rows show what state (0, 1, or 2) the individual was in at a particular time (column position). For the topmost hypnogram, the individual started in state 0, then transitioned to state 1 and remained there for two consecutive periods, then transitioned to state 2 and remained for four consecutive periods before transitioning to her final recorded state 1. Similarly, history rows were constructed for the remaining two hypnograms. These history rows are then stacked into a history matrix. The history matrix itself can be visualized, as can various sortings of the history matrix. In Figure 1, a vertical sorting is performed and then visualized.

All transitional data following many subjects over time can be represented by a history matrix $H$. $H$ is a $m \times n$ matrix where the $m$ rows are the number of subjects in the group and $n$, the number of columns, is representative of the maximum number of intervals of recording of all groups. Therefore, an element $h_{ij}$ of $H$ (where $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$) is a number that corresponds to the state of the $i^{th}$ individual during the $j^{th}$ interval. In this conceptualization, the matrix $H$ is merely a stack of rows depicting a group of individuals’ paths over time.

Transforming the matrix containing numbers into a graphical visualization via “painting by number” is quite useful in exploratory data analysis. Keeping in mind that the underpinning of the graphical figure is a matrix, we broaden the scope of information that can be obtained through simple summary statistics as well as visualization. The crux of this paper is this: the history matrix $H$ itself can be imaged, as can various sortings of $H$. Such sorting allows researchers to glean many important attributes of the data which can guide research directions and aid in gaining understanding of the phenomena.
2. Motivating Example: Apneics vs. Non-Apneics

Sleep architecture is a type of transitional phenomenon in that during sleep an individual can be in a state of wake, stage 1, stage 2, stage 3, stage 4, or REM sleep. From any of these states a transition is possible to any of the five other states. For simplicity and without loss of generality, we will consider the case of 3 states of sleep by collapsing stage 1-4 sleep into Non-REM sleep, leaving the possible states of sleep being: Wake, Non-REM sleep, and REM sleep. Sleep apnea is a condition where the esophagus collapses and suffocates the apneic to the point of awakening. Apneics are defined as having an apnea-hypopnea index of $\geq 30$ events/hr, non-apneics strictly 0.

We will analyze the sleep structure of 59 apneics and 59 non-apneics, matched on age, gender, race, and BMI. From sleep onset, their sleep paths were measured in a three state (Wake, Non-Rem, REM) paradigm every 30 seconds until the individual elected to stop the sleep session (and therefore entered an absorbing state in which data collection ceased). A state was recorded per individual per epoch. The collection of such states per individual over time was put into a row vector and then row vectors were stacked, yielding a history matrix $H^A$ for apneics, $H^N$ for non-apneics. The visualization of each history matrix is stacked to aid comparison within matched set and between groups (Figure 2).

“Above all else show the data.” (Tufte, 2001)

Figure 2 is the data. The vertical axis not only denotes each individual within its group, but denotes the row-ordered matched sets of individuals. For example, individual 2 (row 2) of the apneics is the matched partner of individual 2 (row 2) of the non-apneics. Figure 2 saliently depicts apneics experiencing more short-term and long-term bouts of wakefulness and transitioning more often than non-apneics. The differences between groups in wakefulness and transitioning confirm what any sleep physician knows: the sleep of an apneic is more interrupted than that of a non-apneic.

Sorting within the rows of $H^A$ and $H^N$ would preserve the distinction of individuals within each group and the row-order matching of those individuals between groups, but would collapse the data over time. Conducting a sort within rows and visualizing gives us Figure 3. This visual result is not a display of each individual’s proportionate state time during their personal sleep session. This is because visually the “denominator” would be
the total number of epochs, which is the maximum number of epochs of recording of one particular individual (however, the matrix sorting result does allow easy computation of proportionate state time for each individual.) Figure 3 is useful in the sense that having a certain state/color flush left against the vertical axis allows a viewing of the number of epochs spent in that state, and it preserves the row-order matching between groups. However, getting a sense for the group behavior is more difficult with the row-order matching intact. Enter Strand sorting to facilitate the exploration of group person-time densities of each stage.

Strand sorting the matrix represented in Figure 3 is simply a sorting of *entire* rows. With the state of interest flush left, entire rows are sorted by the number of epochs of the flush left state within each group. This collapses over the row-order matching (if we do not keep track of match numbers) but preserves individual row distinction. The result of Strand sorting for WAKE (Figure 4) renders a more coherent look at the differences between group person-time densities. Strand sorting on ABSORBED with a certain state flush right would enable the exploration of correlations between time in that state and total sleep time.

From $H^A$ and $H^N$ we have conducted Horizontal sorting and Strand sorting, which collapsed over time and preserved individual data. Sorting each $H^A$ and $H^N$ vertically within *column* collapses over individual and preserves time (Figure 5). Here, the vertical axis can be thought of as a percentage of the group in the flush bottom state during an epoch.

Vertically sorting $H^A$ and $H^N$ gives another incisive view into the nature of REM and WAKE of apneics and non-apneics. One may have noticed the “lining up” of the durations in REM in Figure 2, but Vertically sorting clearly distinguishes signal strength, and shows an absence of a REM peak just before three hours after sleep onset in apneics as well as the presence of a sizeable REM peak around hour 8 in apneics (possibly the result of restful REM deprivation). Across a vast majority of epochs more apneics than non-apneics are in WAKE.

3. Discussion

All figures have a tremendous data density ($\geq 2252$ numbers/square inch) (Tufte, 2001) which is higher than that of other graphical methods (Huzurbazar, 2005). Note that the order of colors/states in any sorting is only a matter of bookkeeping in the underlying matrix, and therefore arbitrary. This allows getting states of interest flush against an axis to aid visual investigation. Data need not be stratified or matched for visualization. Start times can
be incorporated easily. Not only the stages themselves, but characteristics of the stages (start time of recording, number of certain types of transitions, number of overall transitions, end time of recording) are easily sorted on and visualized. Utilization of the matrix underpinnings makes the calculations of frequencies of transitions and state person-time densities easy. Visualization of $H$ facilitates data quality checks, with easy detection of outliers, missing data, and appropriate temporal order. Excessive numbers of states, large sample sizes, and/or small epoch sizes threaten the resolution of the visualization of $H$. However, zoom features lessen the extent of this threat. Strand and Vertical sorting would still be useful regardless.

With regard to sleep architecture, research currently predominantly uses composite summary measures that collapse the data over time. An example of this would be “apneics spent 18% of their total sleep time in REM whereas non-apneics spent 21%.” An interesting point is how similar the two groups look using the time-collapsed measures in the Horizontal and Strand sorting as opposed to the discrepancy when viewing the visualizations of the history matrices (Figure 2) and the Vertical sorting (Figure 5). In fact, paired t-tests comparing the total sleep time and percentage of total sleep time spent in Wake, Non-REM, and REM showed no significant differences between apneics and non-apneics. However, the differences revealed by the HMV technique in REM periodicity along with the greater tracks of wakefulness and overall transitioning frequency may help explain why apneic sleep is less restful than non-apneic, even when the composite summary measures depict congruent sleep architectures; thus demonstrating the importance of the HMV technique when dealing with longitudinal categorical data.

The HMV technique serves as the impetus for developing models that account for number of transitions, time to transition, and periodicity of states to better classify sleep architecture within various groups and detect differences of sleep architecture between groups. Such development is in progress (Swihart, et al, 2006).

4. Supplementary Materials

MATLAB code available online: http://www.biostat.jhsph.edu/~bcaffo/downloads.htm
References:


Figure 1: The HMV Process

Extract → Stack → Visualize

Extract Stack Visualize

01122221 → 01122221 → 01122221
0122101 → 01222101 → 01112210
01112210 → 01112100 → 01122211 → 01222221
Figure 2: HMVs for Apneics and Non-apneics
Figure 3: Horizontally Sorted on Wake HMVs for Apneics and Non-apneics
Figure 4: Strand Sorted on Wake HMVs for Apneics and Non-apneics
Figure 5: Vertically Sorted on REM sleep HMVs for Apneics and Non-apneics