Logistic Regression Analysis for Breastfeeding of Nepalese Children

GLM structure:

- Linear predictor: $\eta_{ij} = \mathbf{x}'_{ij}\, \beta$
- Link function: $\log it(\mu_{ij}) = \eta_{ij}$ where $\mu_{ij} = E(Y_{ij}) = P(Y_{ij} = 1)$
- Variance function: $\mathrm{var}(Y_{ij} \mid \mathbf{x}_{ij}) = v(\mu_{ij}) = \mu_{ij}(1-\mu_{ij})$
- Correlation structure: $corr(Y_{ij}, Y_{ik}) = \rho_{jk} = \rho(\mu_{ij}, \mu_{ik}; \alpha)$

Then the covariance between two responses on the same subject is:

$$\mathrm{cov}(Y_{ij}, Y_{ik}) = \rho_{jk}\sqrt{v(\mu_{ij})v(\mu_{ik})}$$

and the variance-covariance matrix of $\mathbf{Y_I}$ is

$V_i = V_i(\beta,\alpha)) = \mathrm{var}(\mathbf{Y}_i) = A_i^{1/2} C_i A_i^{1/2}$ where

$$A_i = A_i(\beta) = \begin{pmatrix} v(\mu_{i1}) & 0 & \cdots & 0 \\ 0 & v(\mu_{i2}) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & v(\mu_{in_i}) \end{pmatrix} \text{ and } C_i = C_i(\alpha) = \begin{pmatrix} 1 & \rho_{12} & \cdots & \rho_{1n_i} \\ \rho_{2n_i} & 1 & \cdots & \rho_{2n_i} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{n_i1} & \rho_{n_i2} & \cdots & 1 \end{pmatrix}$$

For example, if the correlation structure is

- Exchangeable: $corr(Y_{ij}, Y_{ik}) = \rho_{jk} = \alpha$ and $C_i = \begin{pmatrix} 1 & \alpha & \cdots & \alpha \\ \alpha & 1 & \cdots & \alpha \\ \vdots & \vdots & \ddots & \vdots \\ \alpha & \alpha & \cdots & 1 \end{pmatrix}$
- Exponential: $corr(Y_{ij}, Y_{ik}) = \rho_{jk} = \alpha^{|j-k|}$ and $C_i = \begin{pmatrix} 1 & \alpha & \cdots & \alpha^{n_i-1} \\ \alpha & 1 & \cdots & \alpha^{n_i-2} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha^{n_i-1} & \alpha^{n_i-2} & \cdots & 1 \end{pmatrix}$