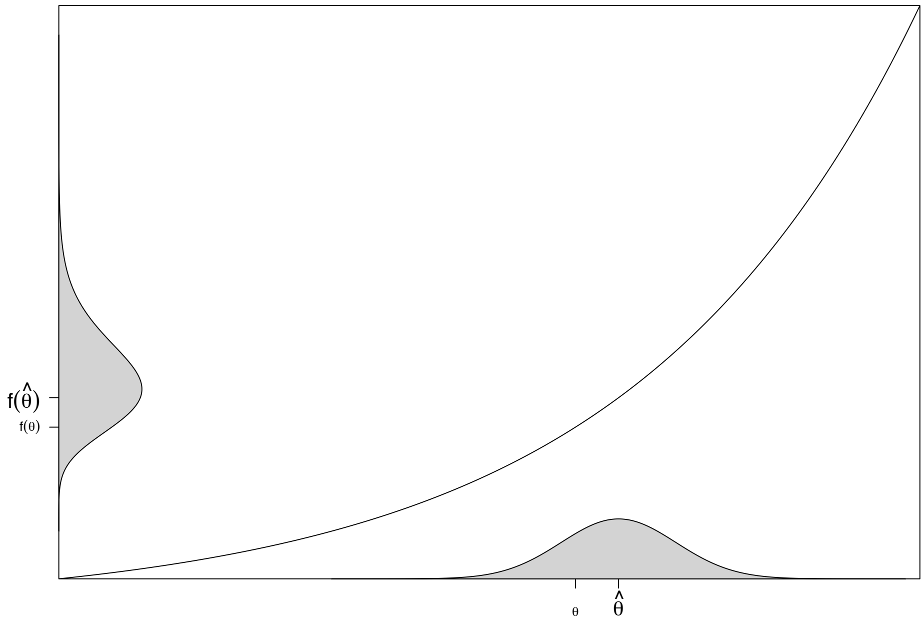
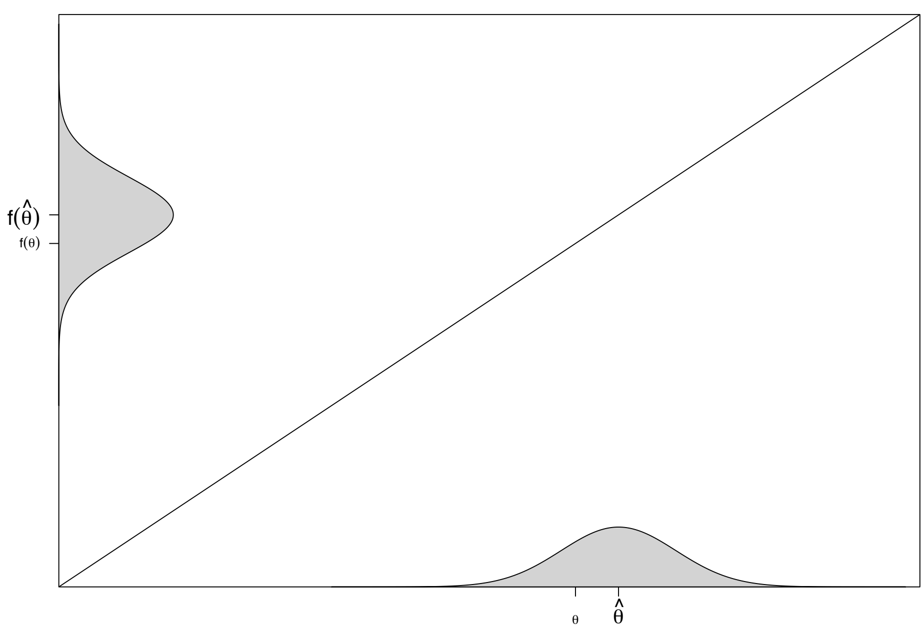
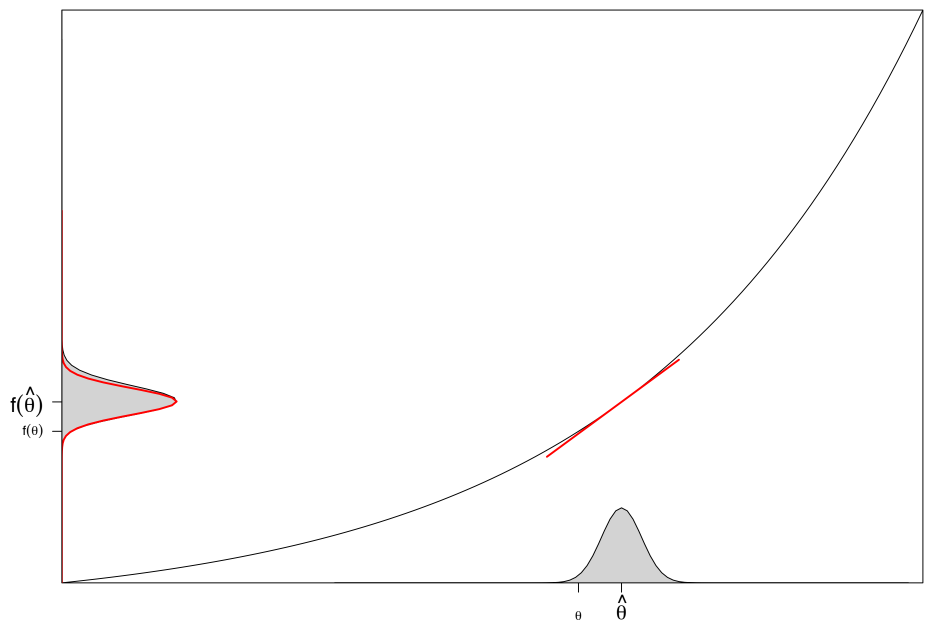
Error Propagation



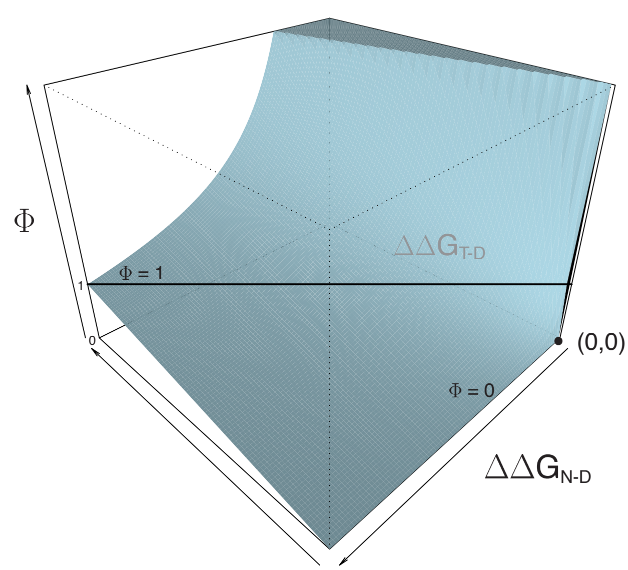
Error Propagation



Error Propagation



Error Propagation



Back to the Sullivan data

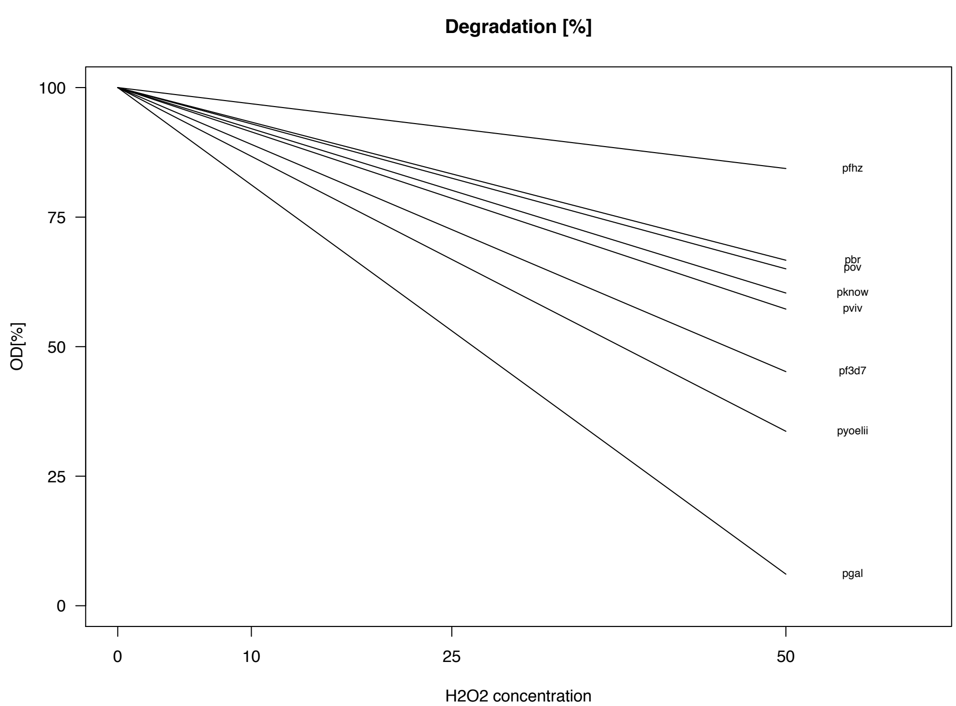
David Sullivan was actually interested in the percent degradation (that is, the slopes when one re-scales the y-axis so that the y-intercept is at 1).

$y = \beta_0 + \beta_1 x + \epsilon$ becomes $y/\beta_0 = 1 + (\beta_1/\beta_0)x + \epsilon'$

So we're really interested in β_1/β_0.

$\longrightarrow$ We'd estimate that by $\hat{\beta}_1/\hat{\beta}_0$, but what about its standard error?

Percent degradation



First-order Taylor expansion

Consider $f(x,y) = x/y$.

A first-order Taylor expansion to approximate the function would be

$$f(x,y) \approx f(x_0,y_0) + (x - x_0)\left.\frac{\partial f}{\partial x}\right|_{(x_0,y_0)} + (y - y_0)\left.\frac{\partial f}{\partial y}\right|_{(x_0,y_0)}$$

Since $\partial f/\partial x = 1/y$ and $\partial f/\partial y = -x/y^2$, we obtain the following:

$$\begin{aligned} x/y &\approx x_0/y_0 + (x - x_0)/y_0 - (y - y_0)x_0/y_0^2 \\ &= (x_0/y_0)[1 + (x - x_0)/x_0 + (y - y_0)/y_0] \end{aligned}$$

How do we use this?

We use the first-order Taylor expansion of $\hat{\beta}_1/\hat{\beta}_0$ around β_1 and β_0.

Variance of a ratio

Remember that β_1 and β_0 are fixed, while $\hat{\beta}_1$ and $\hat{\beta}_0$ are random.

Add the fact that var(X+Y) = var(X) + var(Y) + 2 cov(X,Y)

$$\begin{aligned} \text{var}\{\hat{\beta}_1/\hat{\beta}_0\} &\approx \text{var}\{(\beta_1/\beta_0)[1 + (\hat{\beta}_1 - \beta_1)/\beta_1 + (\hat{\beta}_0 - \beta_0)/\beta_0]\} \\ &= (\beta_1/\beta_0)^2\{\text{var}(\hat{\beta}_1)/\beta_1^2 + \text{var}(\hat{\beta}_0)/\beta_0^2 + 2\,\text{cov}(\hat{\beta}_1,\hat{\beta}_0)/(\beta_1\beta_0)\} \end{aligned}$$

We then replace β_1 and β_0 in this formula with our estimates of them, $\hat{\beta}_1$ and $\hat{\beta}_0$. Further, we replace the variances and covariance with our estimates.

$$\widehat{\text{var}}\{\hat{\beta}_1/\hat{\beta}_0\} = (\hat{\beta}_1/\hat{\beta}_0)^2\{\widehat{\text{var}}(\hat{\beta}_1)/\hat{\beta}_1^2 + \widehat{\text{var}}(\hat{\beta}_0)/\hat{\beta}_0^2 + 2\,\widehat{\text{cov}}(\hat{\beta}_1,\hat{\beta}_0)/(\hat{\beta}_1\hat{\beta}_0)\}$$

The estimated SE is then

$$\widehat{\text{SE}}\{\hat{\beta}_1/\hat{\beta}_0\} = |\hat{\beta}_1/\hat{\beta}_0|\sqrt{[\widehat{\text{SE}}(\hat{\beta}_1)/\hat{\beta}_1]^2 + [\widehat{\text{SE}}(\hat{\beta}_0)/\hat{\beta}_0]^2 + 2\,\widehat{\text{cov}}(\hat{\beta}_1,\hat{\beta}_0)/(\hat{\beta}_1\hat{\beta}_0)}$$

Results

pf3d7:

$\hat{\beta}_0 = 0.353$ (0.005) $\hat{\beta}_1 = -0.0039$ (0.0002) $\hat{\text{cov}}(\hat{\beta}_1, \hat{\beta}_0) = -6.6 \times 10^7$

$\hat{\beta}_1/\hat{\beta}_0 \times 100$ = –1.10 (SE = 0.07).

	estimate	SE
bhem	-2.04	0.32
pgalnoel	-2.02	0.35
pgal	-1.88	0.17
pyoelii	-1.33	0.09
pf3d7	-1.10	0.07
pviv	-0.86	0.26
pknow	-0.79	0.14
pov	-0.70	0.07
pbr	-0.67	0.08
pfhz	-0.31	0.17