Lab 2: Some probability calculations

1 Bayes' Theorem

Suppose there are two events A and B. In probability theory and applications, Bayes' theorem (also known as Bayes' rule or Bayes' law) shows how to get probability of A given B from the conditional probability of B given A. This also involves the so-called prior or unconditional probabilities of A and B.

$$\begin{aligned} P(B) &= P(B \cap A) + P(B \cap A^c) \text{ (Law of total probability)} \\ P(A \cap B) &= P(A|B)P(B) = P(A)P(B|A) \\ P(A|B) &= \frac{P(A \cap B)}{P(B)} \\ &= \frac{P(A)P(B|A)}{P(B)} \text{ (Bayes' rule)} \\ &= \frac{P(A)P(B|A)}{P(A)P(B|A) + P(A^c)P(B|A^c)} \end{aligned} \tag{1}$$

2 Expectation, Variance and Covariance

- $$\begin{aligned} E[aX+b] &= aEX + b \\ E[X+Y] &= EX + EY \end{aligned} \tag{2}$$

- $$\begin{aligned} Cov(X,Y) &= E\big[(X-EX)(Y-EY)\big] \\ Var(X) &= Cov(X) = E(X-EX)^2 \geq 0 \\ Var(aX+b) &= a^2 Var(X) \\ Var(X+Y) &= Var(X) + Var(Y) + 2Cov(X,Y) \end{aligned}$$

- $\rho_{XY} = \pm 1 \Longleftrightarrow Y = aX + b \quad a.s.$
  $\Leftarrow$ Easy.
  $\Rightarrow$ $\rho_{XY} = 1$ if and only if $Var(X/\sigma_X - Y/\sigma_Y) = 0$.