Lab 2: Some probability calculations

1 Bayes’ Theorem

Suppose there are two events $A$ and $B$. In probability theory and applications, Bayes’ theorem (also known as Bayes’ rule or Bayes’ law) shows how to get probability of $A$ given $B$ from the conditional probability of $B$ given $A$. This also involves the so-called prior or unconditional probabilities of $A$ and $B$.

\[
\begin{align*}
P(B) &= P(B \cap A) + P(B \cap A^c) \text{ (Law of total probability)} \\
P(A \cap B) &= P(A|B)P(B) = P(A)P(B|A) \\
P(A|B) &= \frac{P(A \cap B)}{P(B)} \\
    &= \frac{P(A)P(B|A)}{P(B)} \text{ (Bayes’ rule)} \\
    &= \frac{P(A)P(B|A)}{P(A)P(B|A) + P(A^c)P(B|A^c)} \\
\end{align*}
\]

2 Expectation, Variance and Covariance

- \[E[aX+b] = aEX+b\]
- \[E[X+Y] = EX+EY\] (2)

- \[\text{Cov}(X,Y) = E[(X-EX)(Y-EY)]\]
- \[\text{Var}(X) = \text{Cov}(X) = E(X-EX)^2 \geq 0\]
- \[\text{Var}(aX+b) = a^2\text{Var}(X)\]
- \[\text{Var}(X+Y) = \text{Var}(X) + \text{Var}(Y) + 2\text{Cov}(X,Y)\]

- $\rho_{XY} = \pm 1 \iff Y = aX + b \quad \text{a.s.}$
  \[\Leftarrow \text{Easy.}\]
  \[\Rightarrow \rho_{XY} = 1 \text{ if and only if } \text{Var}(X/\sigma_X - Y/\sigma_Y) = 0.\]