Solution to Homework 2

Question 1

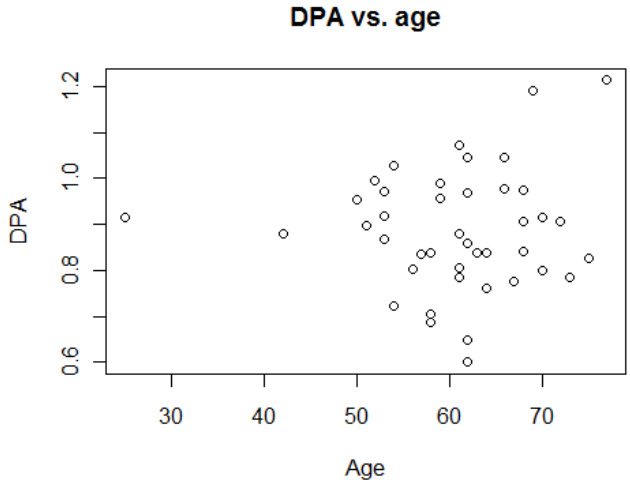
(a). The test statistic $T = \frac{\mu_1 - \mu_0}{\sqrt{s_1^2/n_1 + s_0^2/n_0}}$. For the data we have, we got $T = -3.095 < -1.96$. Then the null hypothesis is rejected. DPA measurement is associated with osteoporosis status. The 95% confidence interval is $(-0.19, -0.04)$.

(b). We have at least 95% confidence to claim that there is association.

(c). The p-value is $P(|T| \geq 3.095) = 0.002$. i.e., If there is no association between the DPA and osteoporosis, there is only a probability of less than 0.002 to get the test statistic value as extreme as what we got from our data.

(d). The mean difference in DPA between $osteo = 1$ and $osteo = 0$ groups is estimated to be -0.114. The 95% level confidence interval of this mean difference is in $(-0.19, -0.04)$. The DPA for $osteo = 1$ group is significantly lower than that in the $osteo = 0$ group with p-value 0.002.

Question 2



The sample correlation coefficient is 0.073. It is 0.384 in the $osteo = 0$ group, but only -0.0003 in the $osteo = 1$ group. The linear correlation between DPA and age is minor. In fact, there are some association in the $osteo = 0$ group. But DPA and age are almost uncorrelated in the $osteo = 1$ group.

Question 3

Answer (c).
(a) and (b) are talking about the linear correlation. Independence implies uncorrelated, but not true versa; (d) is right if it is 'equal'.

Question 4

(a). $Pr(user|positive) = 25\%$

$$\begin{aligned} Pr(user|+) &= \frac{Pr(user \cap +)}{Pr(+)} \\ &= \frac{Pr(user)Pr(+|user)}{Pr(user)Pr(+|user) + Pr(nonuser)Pr(+|nonuser)} \\ &= \frac{1}{1 + \frac{Pr(nonuser)Pr(+|nonuser)}{Pr(user)Pr(+|druguser)}} \\ &= \frac{1}{1 + \frac{95\%*15\%}{5\%*95\%}} \\ &= 25\% \end{aligned} \tag{1}$$

(b). $Pr(nonuser|negative) = 99.7\%$

(c). This test does not do very well in picking true users among those with positive test results. If we look at the formula in (a), it is because $\frac{Pr(nonuser)}{Pf(user)}\frac{Pr(+|nonuser)}{Pr(+|user)}$ is large. Since $\frac{Pr(nonuser)}{Pf(user)}$ are the percentage in the population and is fixed. We cannot do much on this. So if we can increase $Pr(-|nonuser)$ or $Pr(+|user)$, then $Pr(user|+)$ will increase.

Question 5

Answer (a).
The population mean is a fixed number anyway. The confidence level of CI is saying the percentage that the CI's covering the mean in repeated experiment.

Question 6

Answer (e).
Type I error is the probability of rejecting the null when the null hypothesis is true.