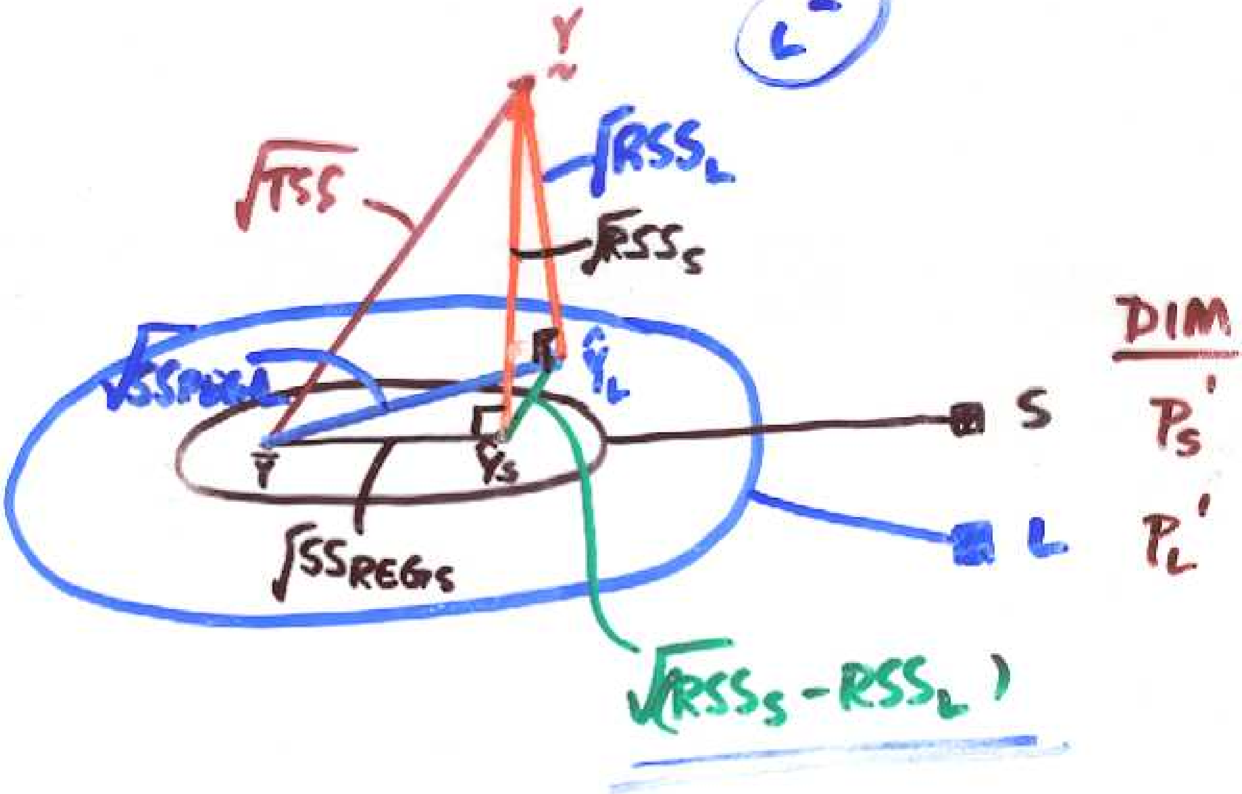
BIOS 653



IMPORTANT FEATURES

1) Orthogonality "$\perp$" $\Rightarrow$

a) $SS_{REG_L} = TSS - RSS_L$

$+$ $SS_{REG_S} = TSS - RSS_S$

$$\boxed{\Rightarrow SS_{REG_L} = SS_{REG_S} + RSS_S - RSS_L}$$

$\perp$

b) POSS. TO ROTATE (PERPENDICULAR) COORDINATE AXES SO THAT

- 1st P_S' AXES $\in$ S
- NEXT $P_L' - P_S'$ AXES $\in$ L (not S)
- FINAL $n - P_L'$ AXES $\in$ $L^{\perp}$

achieved by $\tilde{Y} \Rightarrow T\tilde{Y}$

$\underset{n \times n}{T}\tilde{Y}$

orthogonal

$TT' = I$

IMPORTANT FEATURES, CONTD.

2) INDEPENDENT NORMAL DECOMPOSITION

(a) $T\underset{\sim}{Y} = \underset{\sim}{z} \sim N(T\mu_Y, \underbrace{\sigma^2 TT'}_{=\sigma^2 I})$

$T'T\underset{\sim}{Y} = T'\underset{\sim}{z}$

$\Rightarrow$ (b) $\underset{\sim}{Y} = T'\underset{\sim}{z}$

$$= \underbrace{\sum_{i=1}^{p_s'} z_i \underset{\sim}{e_i}}\ + \underbrace{\sum_{i=p_s'+1}^{p_L'} z_i \underset{\sim}{e_i}}\ + \underbrace{\sum_{i=p_L'+1}^{n} z_i \underset{\sim}{e_i}}$$

with z_i mutually independent normal $\bar{w}$ variance σ^2

e_i is the i^{th} column of T

(c) Under $H_0: E[\underset{\sim}{Y}] \in S$

$E[z_i] = 0, \quad i > p_s'$

(d) $\hat{Y}_S = \sum_{i=1}^{p_s'} z_i \underset{\sim}{e_i}$ (b/c $H_S \underset{\sim}{e_i} = \underset{\sim}{e_i}$, $i \le p_s'$

$= 0$ ow)

$\hat{Y}_L = \sum_{i=1}^{p_L'} z_i \underset{\sim}{e_i}$

(e) $RSS_L = \sum_{i=1}^{n} (\underset{\sim}{Y_i} - \hat{Y}_i)^2 = \sum_{i=p_L'+1}^{n} z_i^2 \sim \sigma^2 \chi^2_{n-p_L}$

$\underline{RSS_S - RSS_L} = \sum_{i=p_s'+1}^{p_L'} z_i^2 \sim \sigma^2 \chi^2_{p_L - p_S}$

SUMMARY

FROM (c), under H_0

- $RSS_L \sim \sigma^2 \chi^2_{n-p_L}$
- $RSS_S - RSS_L \sim \sigma^2 \chi^2_{p_L - p_S}$

INDEP.

THEREFORE,

$$\frac{(RSS_S - RSS_L)/(p_L - p_S)}{RSS_L/(n-p_L')} \quad \left\{ \frac{\chi^2_{p_L-p_S}/(p_L-p_S)}{\chi^2_{n-p_L'}/(n-p_L')} \right. \quad \text{indep}$$

SATISFIES THE DEFINITION OF AN F - random variable with $p_L - p_S$, $n - p_L$ d.f.

FINAL POINT: DECOMPOSITION SUMMARIZES WHAT "DEGREES OF FREEDOM" MEANS.

WHY F-test?
1) CONVENIENT DISTRIBUTION
2) FACE VALIDITY
3) $\Leftrightarrow$ LIKELIHOOD RATIO TEST